Tackling the profusion vs novelty issue : Punctuated evolution by simple inflation of a random matrix

H. Benisty[1,2]

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and

[2] *LIED, Université de Paris, Paris, (13ème)*

(LIED : Lab Interdisciplinaire des Energies de Demain)

OUTLINE

- **Growth, its extreme, its « noise »**
- My toy in photonics : « PT Symmetry » ⇔ Non-Hermitian H with real eigenvalues
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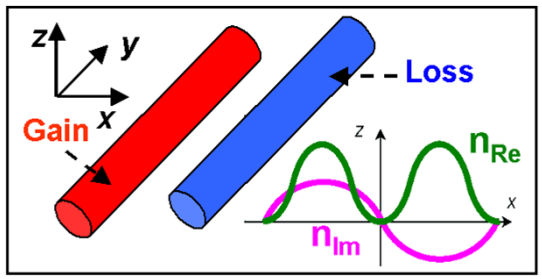
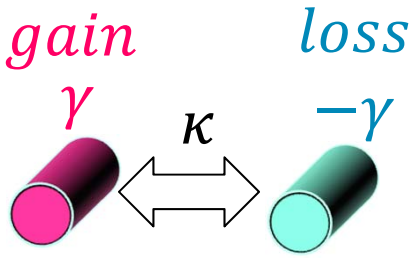
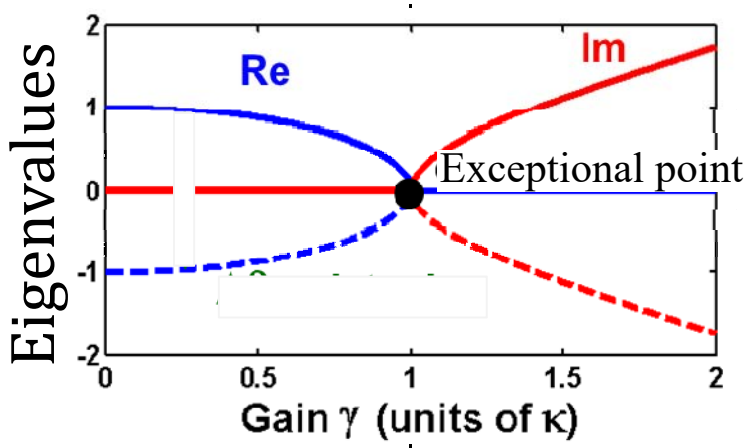
Gain, loss, stability with model Non-Hermitian systems (1/11)





κ : coupling

$$\frac{d}{dt}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} i\gamma & \kappa \\ \kappa & -i\gamma \end{pmatrix}\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$



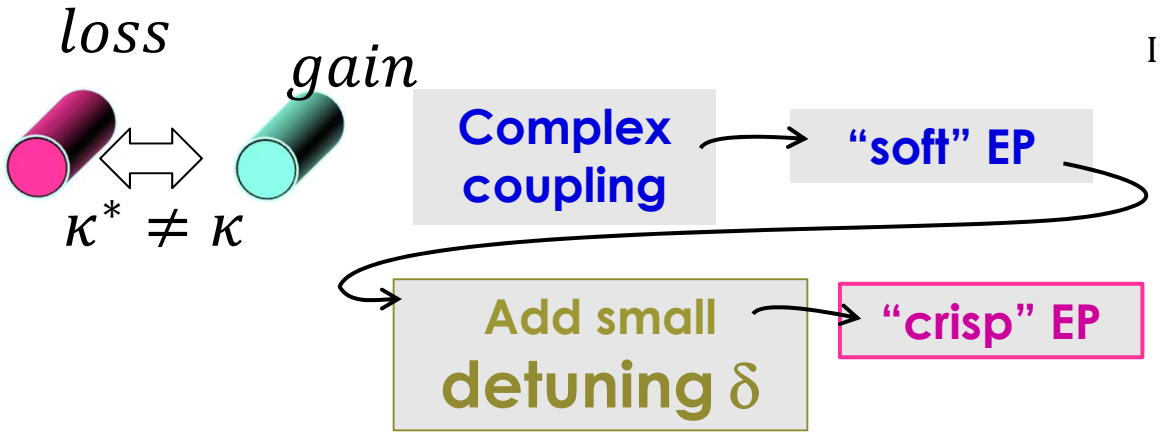
unbroken phase | broken phase

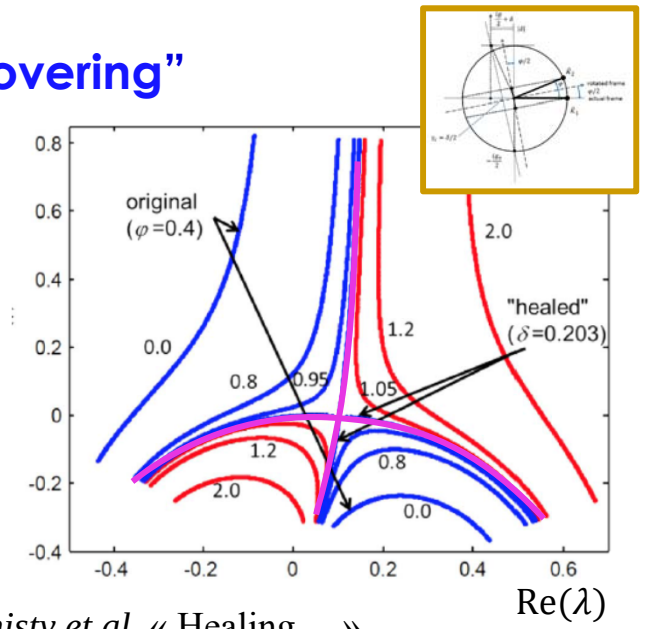
Real eigenv. *Conjug. eigenv.*

$A_{1,2}$? "*Amplitudes*"
E-field (electromag)
Photon numbers ?
$|\psi\rangle$ (of anything)
[Enzyme/chem species] concentration ?

Example of some games in town (1/11)

•• Exceptional points and "EP healing/recovering"

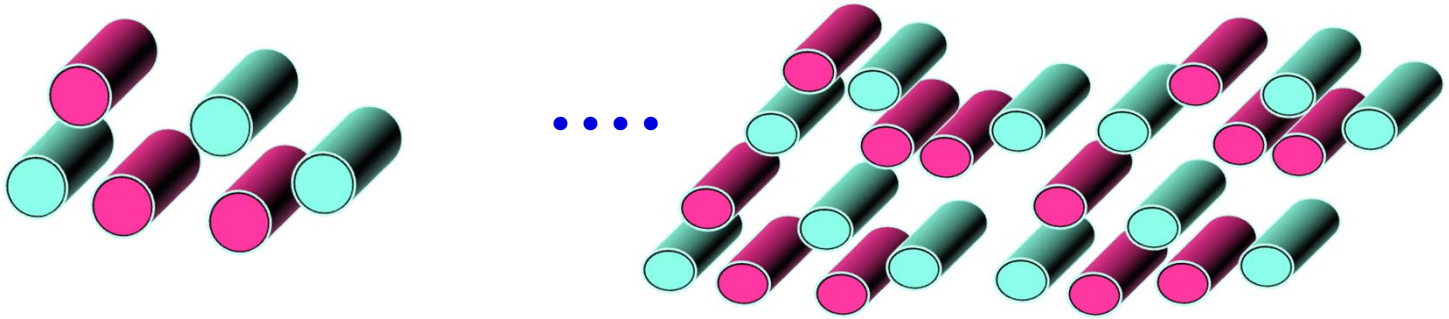




IEEE JLT 2012, *Benisty et al.* « Healing …»
New J. Phys. 18 (2016) 125012, *Nguyen et al.* "Recovering …"

••• what about more general cases ?

(~ generalization of "tranverse PT symmetry" PRA 2015 H. Benisty + EPJB 2020 (2020) 93: 192)



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A first landing in stochastic-land

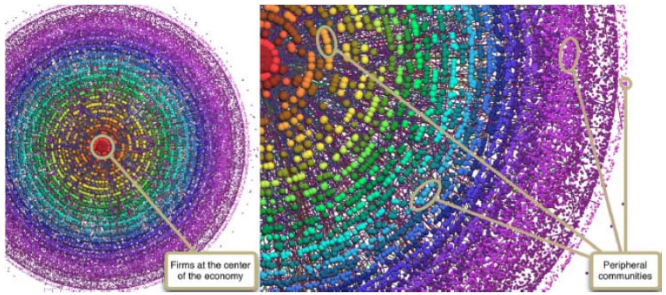
« novelty 1.0 : crisis ? »

« the tail wags the dog »

- Initial interest :
  2008 crash / growth & collapse of option-based assets ?
  Crisis amplified from « small » agents ?
  (a few millions poor owners in USA)

- Simple way : multi-agent models (à la Axtell,…)

Employment Growth through Labor Flow Networks
Omar A. Guerrero, Robert L. Axtell



PloS One May 2013 | Volume 8 | Issue 5 | e60808

- **Even simpler way … ?!**
  → almost independent agents …

Geometric Brownian Motions

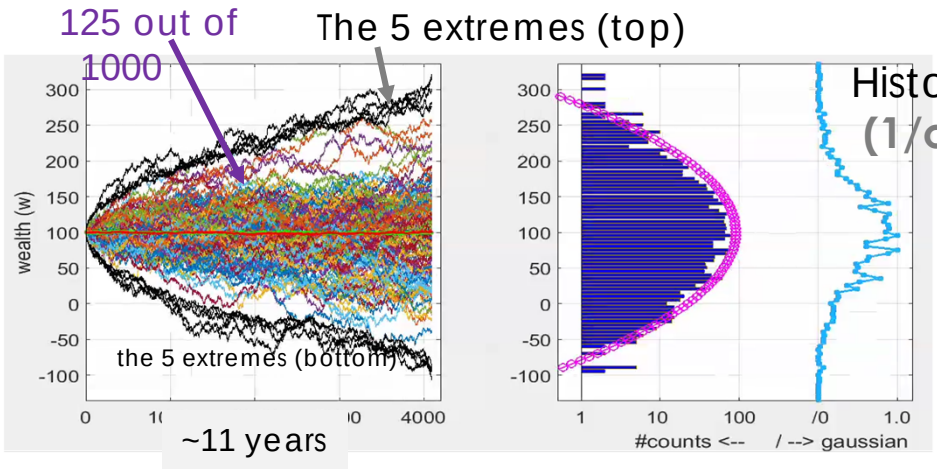
• Geom. vs. Arithm.

(econophysicists)

• Arithmetic

1000 Random Walks
wealth(t_0)=100,

• Additive Noise= ±1/day

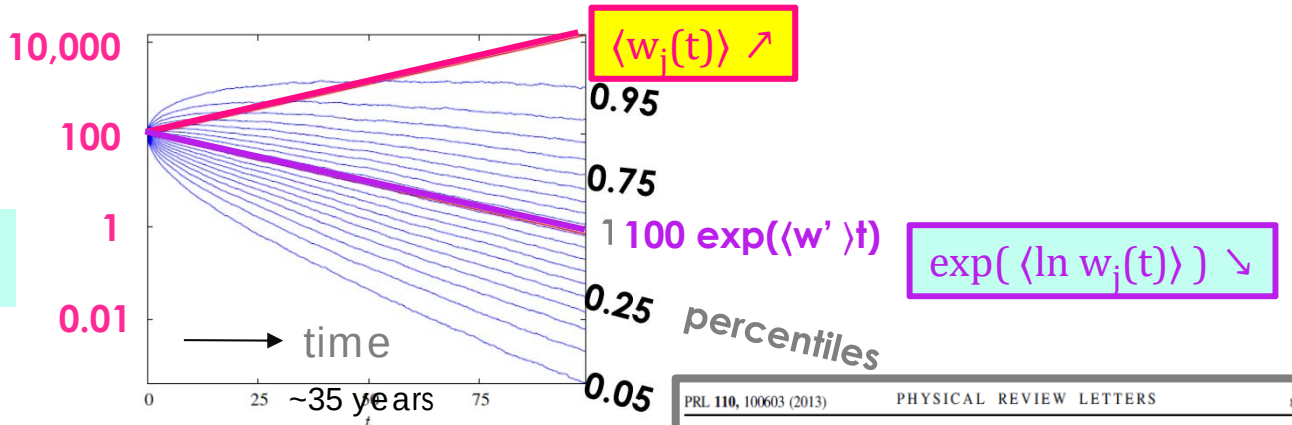


Histogram = Gaussian
$(1/\sigma) \times \exp(-\Delta x^2/2\sigma^2)$

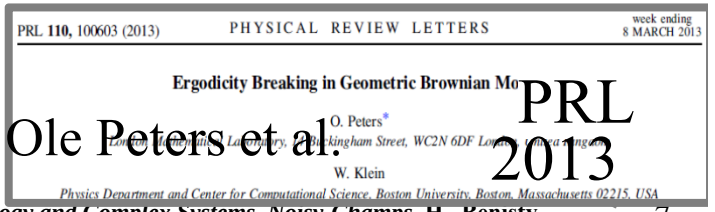
• Geometric : « just » the exponential of those random walks…

• Geom. noise :
[×1.01 / × 0.99] / day

1.01×0.99=0.9999<1



→ **Non stationary ensemble average (diverges) → Non-ergodicity**


PRL 110, 100603 (2013) PHYSICAL REVIEW LETTERS week ending 8 MARCH 2013

Ergodicity Breaking in Geometric Brownian Mo

O. Peters*

W. Klein

Physics Department and Center for Computational Science, Boston University, Boston, Massachusetts 02215, USA

Ole Peters et al. PRL 2013

A citation by Francis Bacon; a paper by Gell-Man

Money is like muck
[⇔ manure],
not good except it be spread.

XV — Of Seditions and Troubles 47

is notably seen in the Low-Countrymen, who have the best mines above ground, in the world.

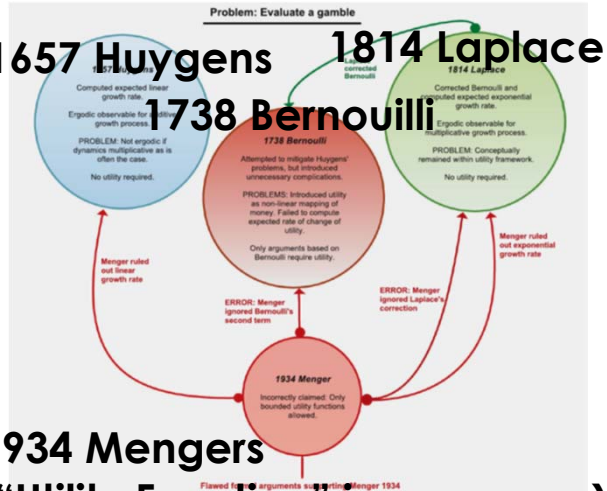
Above all things, good policy is to be used, that the treasure and moneys, in a state, be not gathered into few hands. For otherwise a state may have a great stock, and yet starve. And money is like muck, not good except it be spread. This is done, chiefly by suppressing, or at least keeping a strait hand, upon the devouring trades of usury, ingrossing great pasturages, and the like.

http://www.esp.org/books/bacon/essays/contents/essay15.pdf

CHAOS **26**, 023103 (2016)

Evaluating gambles using dynamics

O. Peters[1,a] and M. Gell-Mann[2,b]



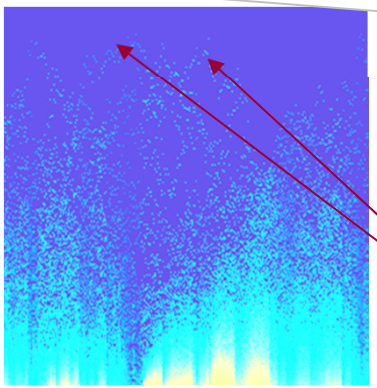
1657 Huygens
1814 Laplace
1738 Bernouilli
1934 Mengers
(wrongly bounds "Utility Function" in economy)

Inequality model without external shocks (HB, 2016-2017, Phys Rev. E)

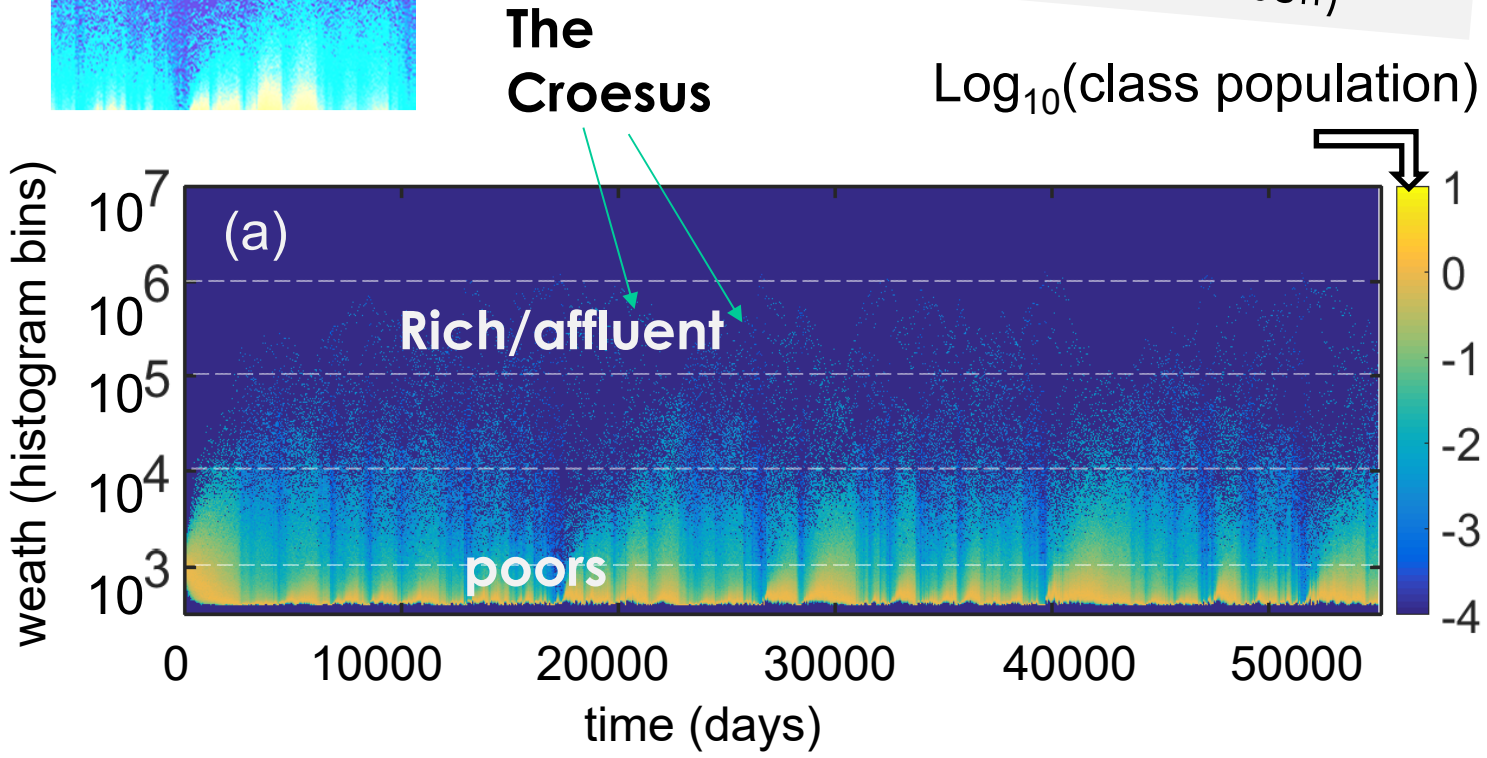
PHYSICAL REVIEW E95, 052307 (2017)

Simple wealth distribution model causing inequality-induced crisis without external shock

Henri Benisty



« the tail wags the dog »
(stylised result)



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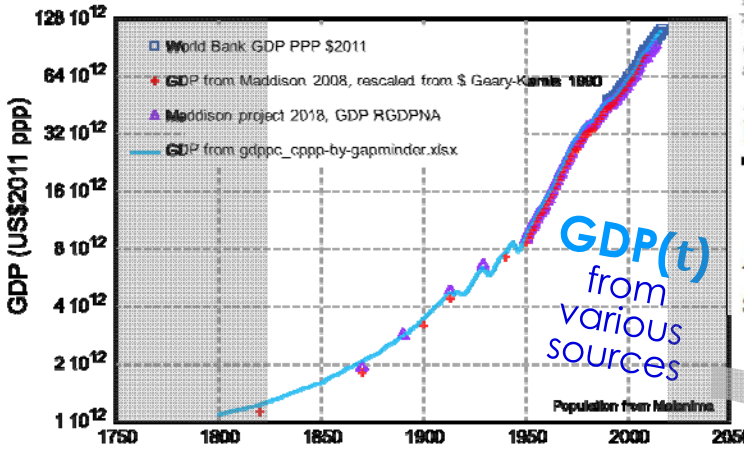
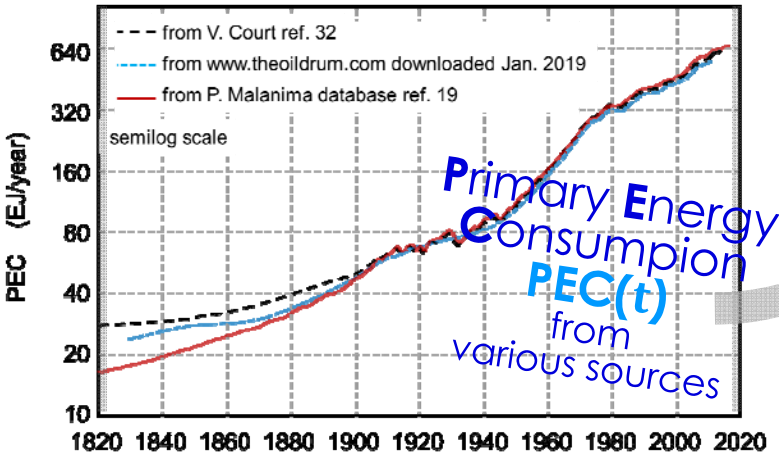
Macro-economic study of {GDP ⇔ energy} relations

Work mostly led by Hervé Bercegol (CEA / LIED)





Ecological Economics 192 (2022) 107253

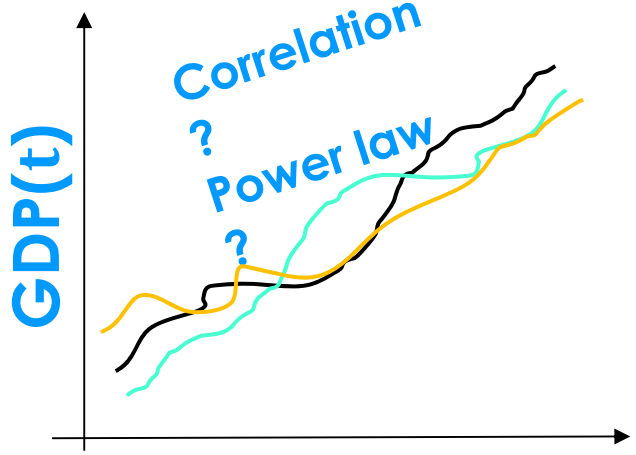
Ecological Economics

journal homepage: www.elsevier.com/locate/ecolecon

ELSEVIER

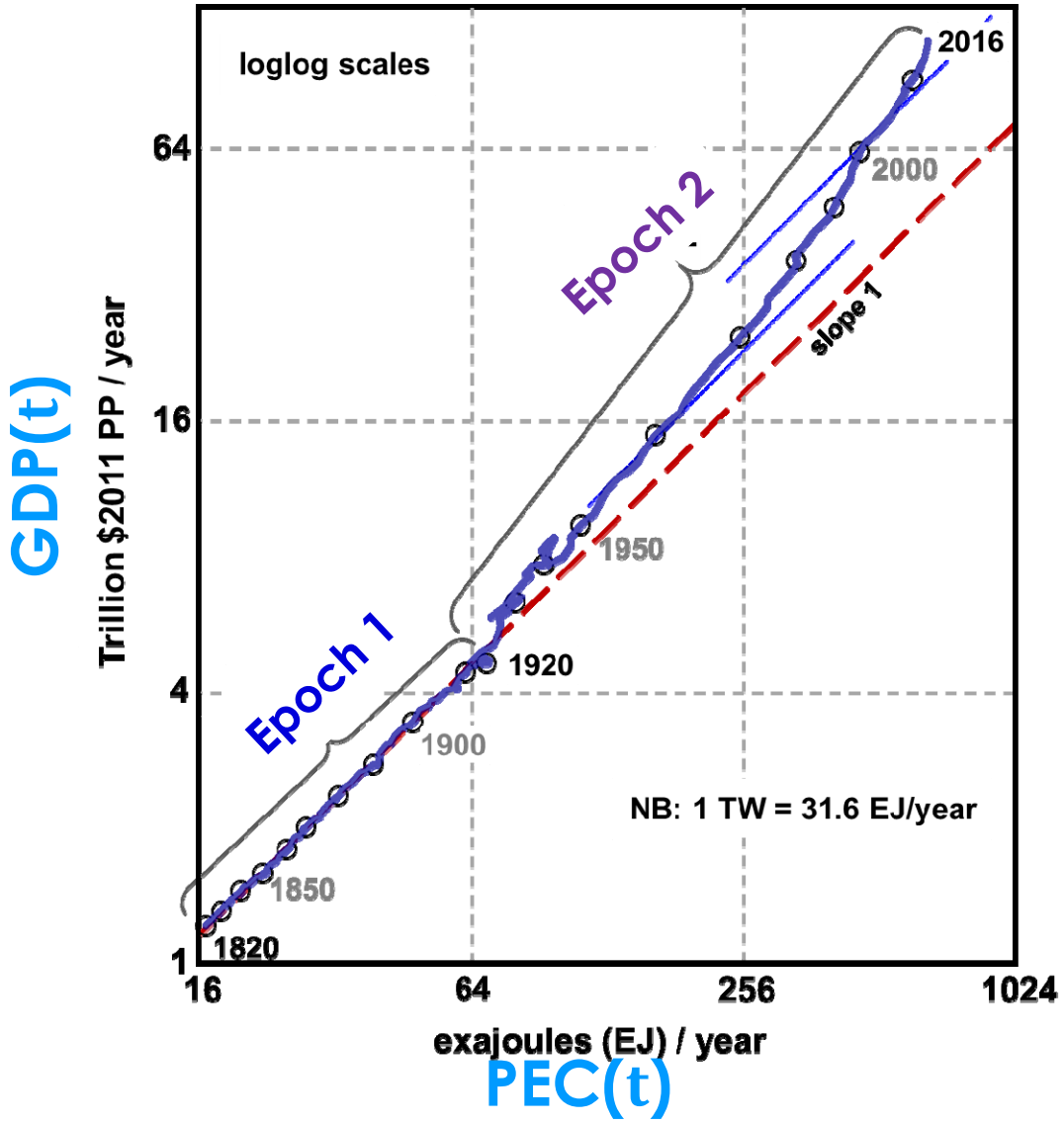
An energy-based macroeconomic model validated by global historical series since 1820

Hervé Bercegol [a,c], Henri Benisty [b,c]



+ Cobb-Douglas, Solomon residual …

Two epochs,
…so far so good for "long term trend"…



- **Epoch 1 (1820-1920)**
  little role of techno/skill
  $GDP \propto PEC$

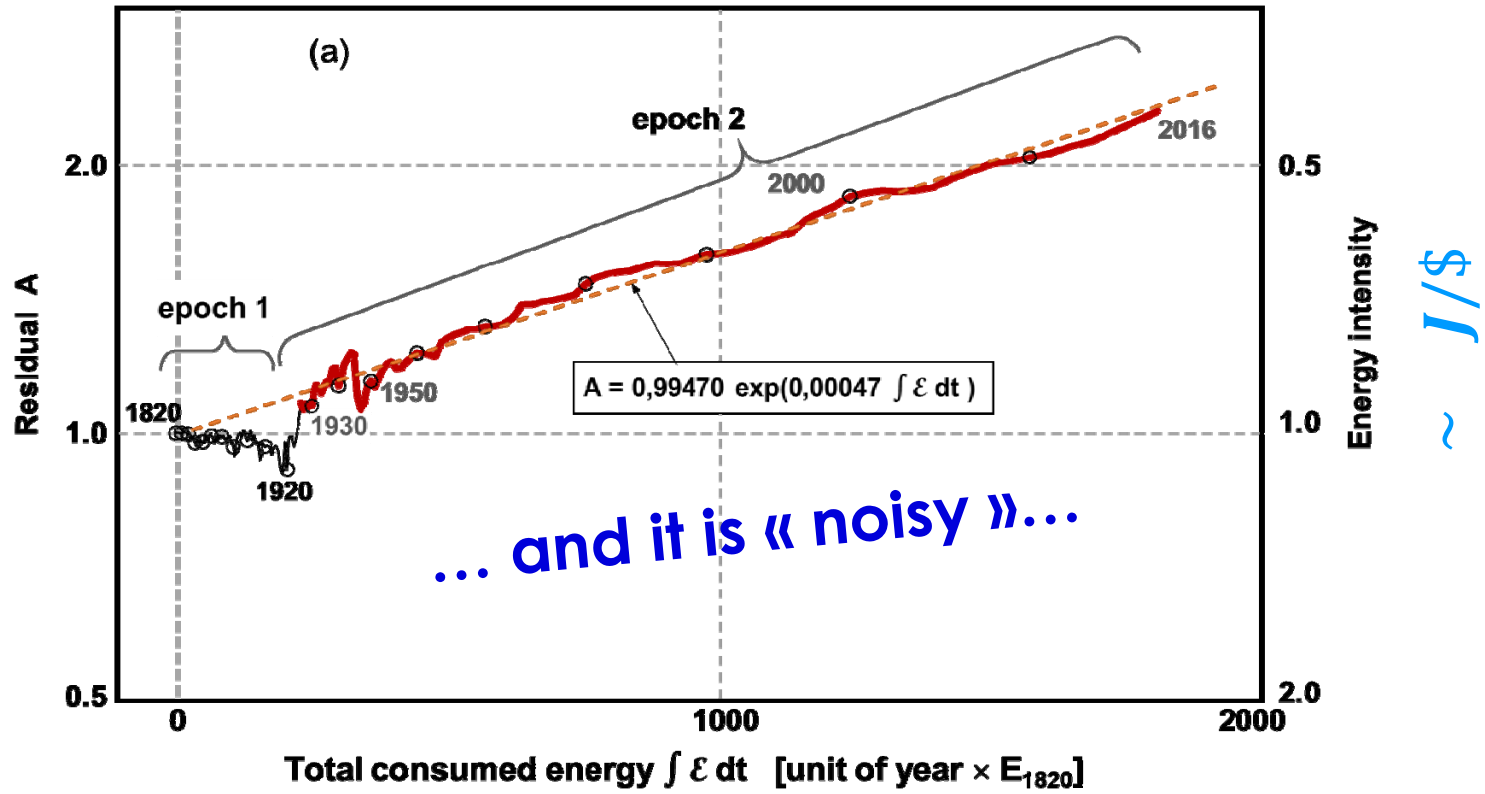
- **Epoch 2 (1920-2016)**
  Cumulated role ~ × 2.5
  $GDP \propto PEC \times \int$ **Cumulated skills**

$$\frac{Y}{Y_{1820}} = exp\left[\int_{1820}^{t} \chi \frac{E}{E_{1820}} \mathrm{dt} \right] \frac{E}{E_{1820}}$$

The « residual » grows mainly in epoch 2

(growth of « residual » seems linear with $\int PEC(t)\,dt$)

GDP/PEC ~ \$/J



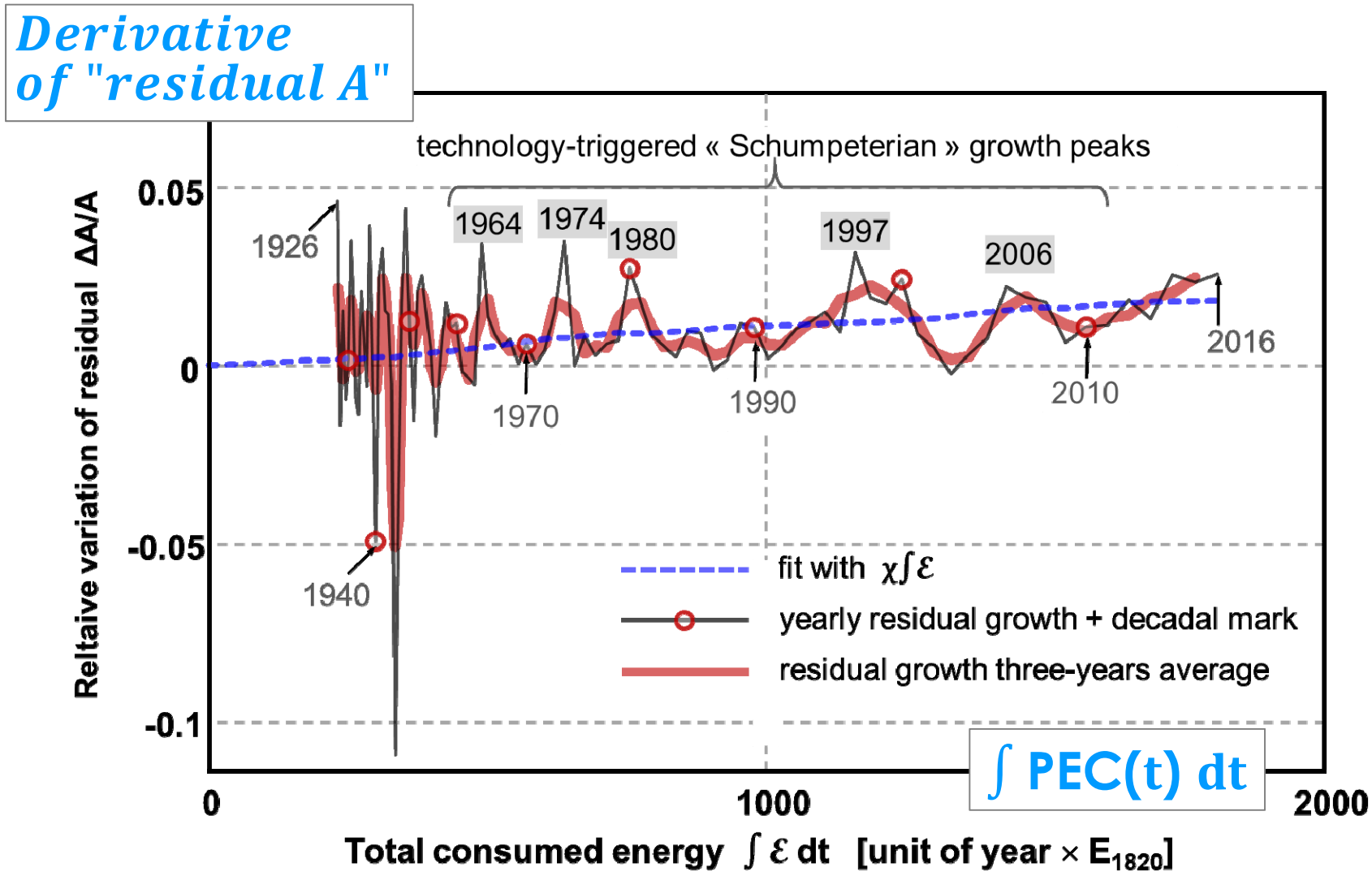
… and it is « noisy »…

J/\$ ~

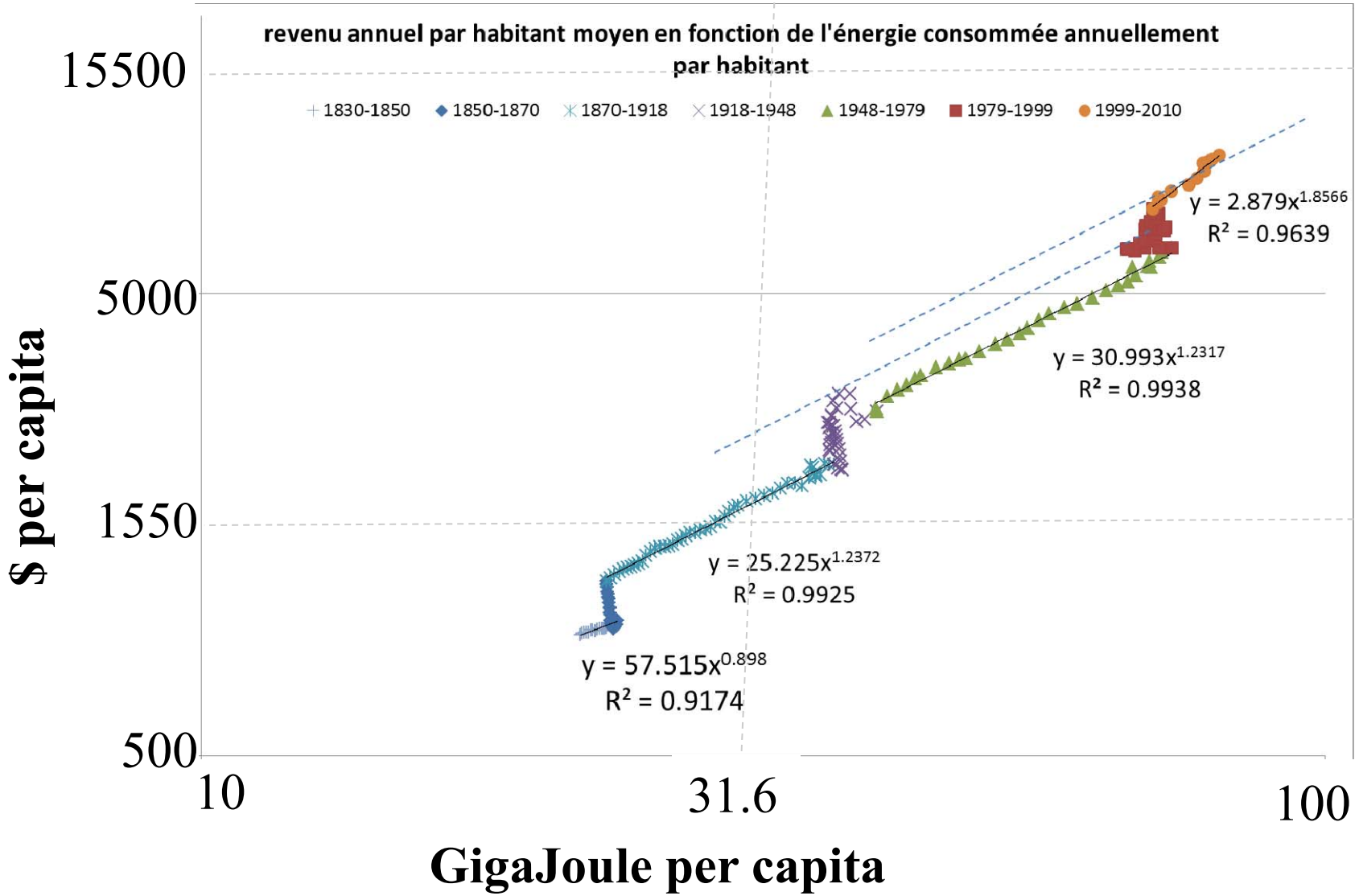
$\int PEC(t)\,dt$

But at short scale data are “quaky” (and “stasy”)

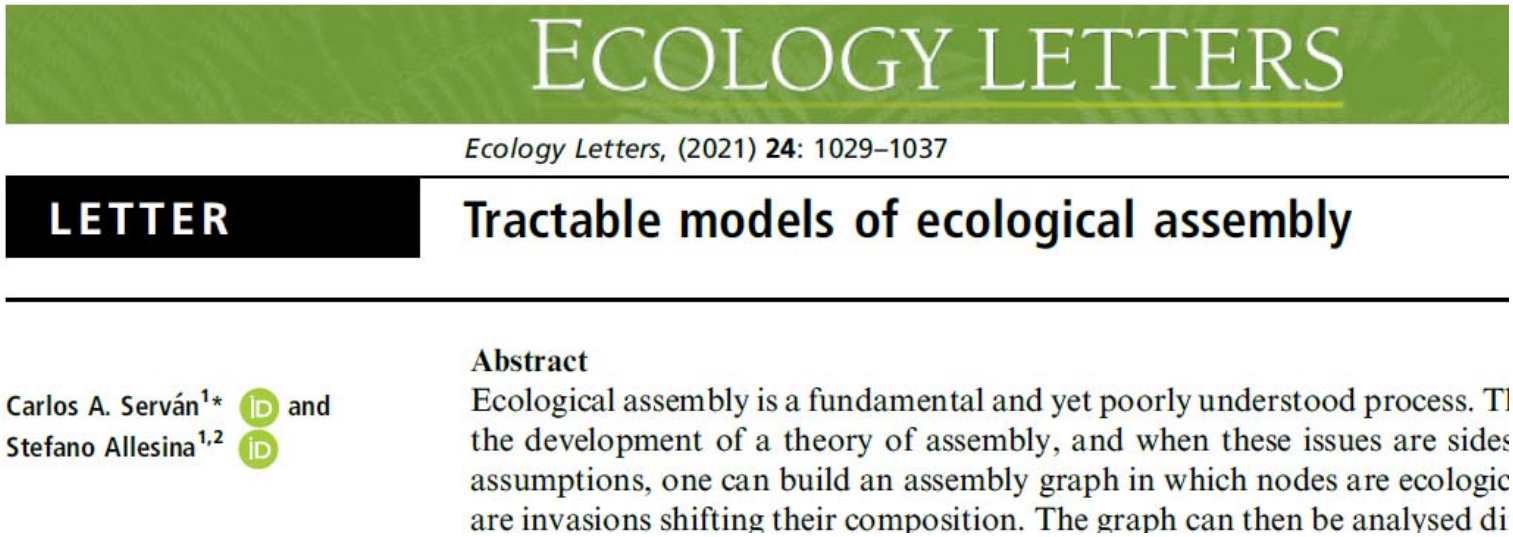
Hard to attribute peaks to a specific techno → “*some systemic thinking is needed*”



Data… has some kind of punctuated growth !



Starting a quest for "profusion of things" in economy (⇔ ? Profusion of species in evo ?)



ECOLOGY LETTERS

Ecology Letters, (2021) **24**: 1029–1037

LETTER

Tractable models of ecological assembly

Carlos A. Serván[1]* and Stefano Allesina[1,2]

Abstract

Ecological assembly is a fundamental and yet poorly understood process. Tl
the development of a theory of assembly, and when these issues are sides
assumptions, one can build an assembly graph in which nodes are ecologic
are invasions shifting their composition. The graph can then be analysed di

How "industry" (at large) "assembles" ?

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Punctuated evolution

Opinion

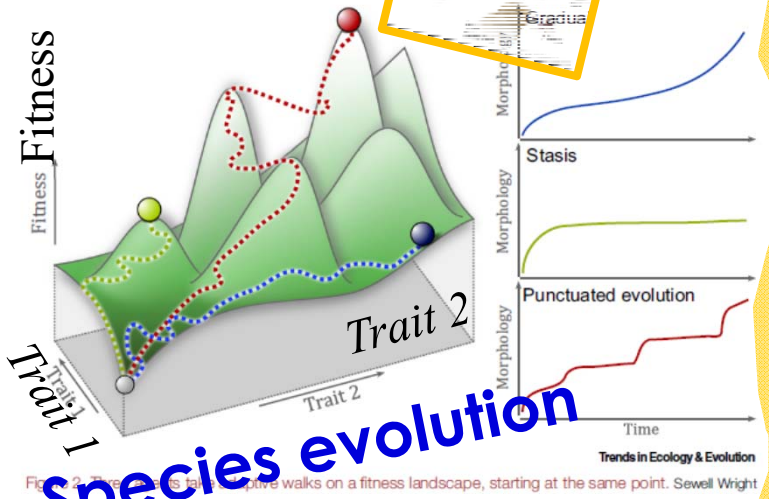
On the multiscale dynamics of punctuated evolution

TREE 3308

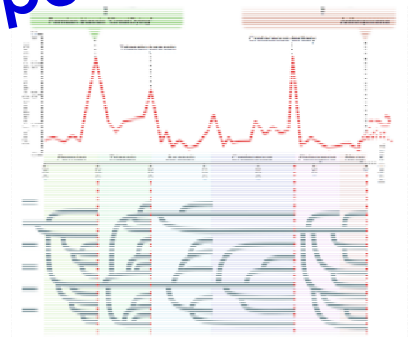
Trends in Ecology & Evolution

Salva Duran-Nebreda [1,@], R. Alexander Bentley [2,@], Blai Vidiella [1,@], Andrej Spiridonov [3,@], Niles Eldredge [4,@], Michael J. O'Brien [5], and Sergi Valverde [1,7,*,@]

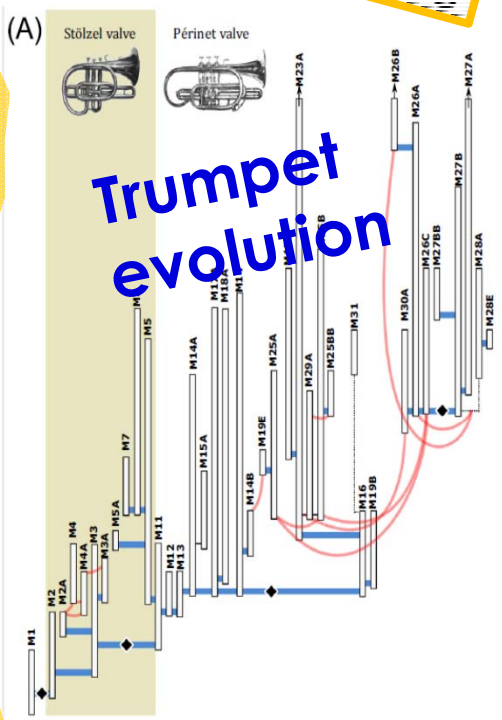
cf. Eldredge & Gould 1972



Species evolution



Trumpet evolution



Coding evolution

Figure I. Phylogenies of cornets and programming languages. (A) The evolution of cornet designs from 1825 onwards is depicted in a figure adapted from Tëmkin

Punctuated « Profusion of elements »

(species or goods)

Certainly there are other perspectives to the issue !!

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Network spandrels reflect ecological assembly

Daniel S. Maynard*[1], Carlos A. Serván[1], and Stefano Allesina[1,2,3]

[1]Department of Ecology & Evolution, University of Chicago,

(from my ~2020 notes)
(I read SJ Gould « S T E »
2000 pages during covid)

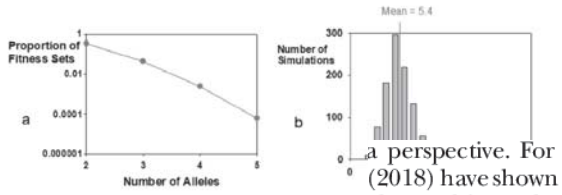


FIGURE 3. AHISTORICAL AND HISTORICAL MODELS OF THE LIKELIHOOD OF
a. The proportion of random sets of constant viabilities that afford full
of alleles (after Lewontin et al. 1978). b. The distribution of the number
constructionist simulation of the single-locus constant-viability model of
see the online edition for a color version of this figure.

a perspective. For instance, Maynard et al. (2018) have shown that a number of patterns observed in ecological networks are not selected directly. Rather they are the incidental consequences of the process of community assembly, so called "spandrels" *sensu* Gould and Lewontin (1979). In an example from population-genetic theory, Trotter and Spencer (2008) showed that sufficiently fit mutant alleles could always invade a population governed by a very general model of frequency-dependent selection. In a sense, then, such populations never reach equilibrium; historical construction is never-ending.

No. 4 *THE QUARTERLY REVIEW OF BIOLOGY* DECEMBER 2020

BEYOND EQUILIBRIA: THE NEGLECTED ROLE OF HISTORY IN ECOLOGY AND EVOLUTION

HAMISH G. SPENCER
Department of Zoology, University of Otago
Dunedin 9056 New Zealand

Punctuated changes and Non-Hermitian Random Matrices (RM) ? (2/10)

- **Main involvment of Random Matrices in Biology/Ecology :**

  R. M. May, 1972 "Can a large system be stable" ?

  "Wigner-May" idea : instability argument based on « circle law » of eigenvalues (Ginibre set [NH] or other)
  " *If coupling too large or too abundant, eigenvalues exceed stability threshold (linearization, Jacobian, …)*".

  (~ still OK if Lotka-Volterra prey-predator dynamics for instance…)

- **Other RM approaches for large systems *rather not tackling* generic "punctuation".**

  (e.g. model agents & innovation in economy)

- **So …**

[2] H. Benisty, "Evolutionary behaviour …", J. Phys. Complex, 3, 025006 (2022).

[3] H. Benisty, "Growth (…) Inflating Complex Random Matrices", Entropy 25, 1507 (2023). https://doi.org/10.3390/e25111507

Punctuated changes and Non-Hermitian Random Matrices (RM) ?

Main simplifying ideas :

- **The basic loops « things → things » or « species → species » can be described by** Vector(t) = Matrix(? t ?) × Vector (t)

- **Attractor replaced by largest module eigenvalue (LMEV) λ_{max} + associated eigenvector. Defines stasis and *dynamics-to-stasis* ($\Delta\lambda = |\lambda_{max}| - |\lambda_{next}|$) (/!\ in Non-Hermitian setting…)**

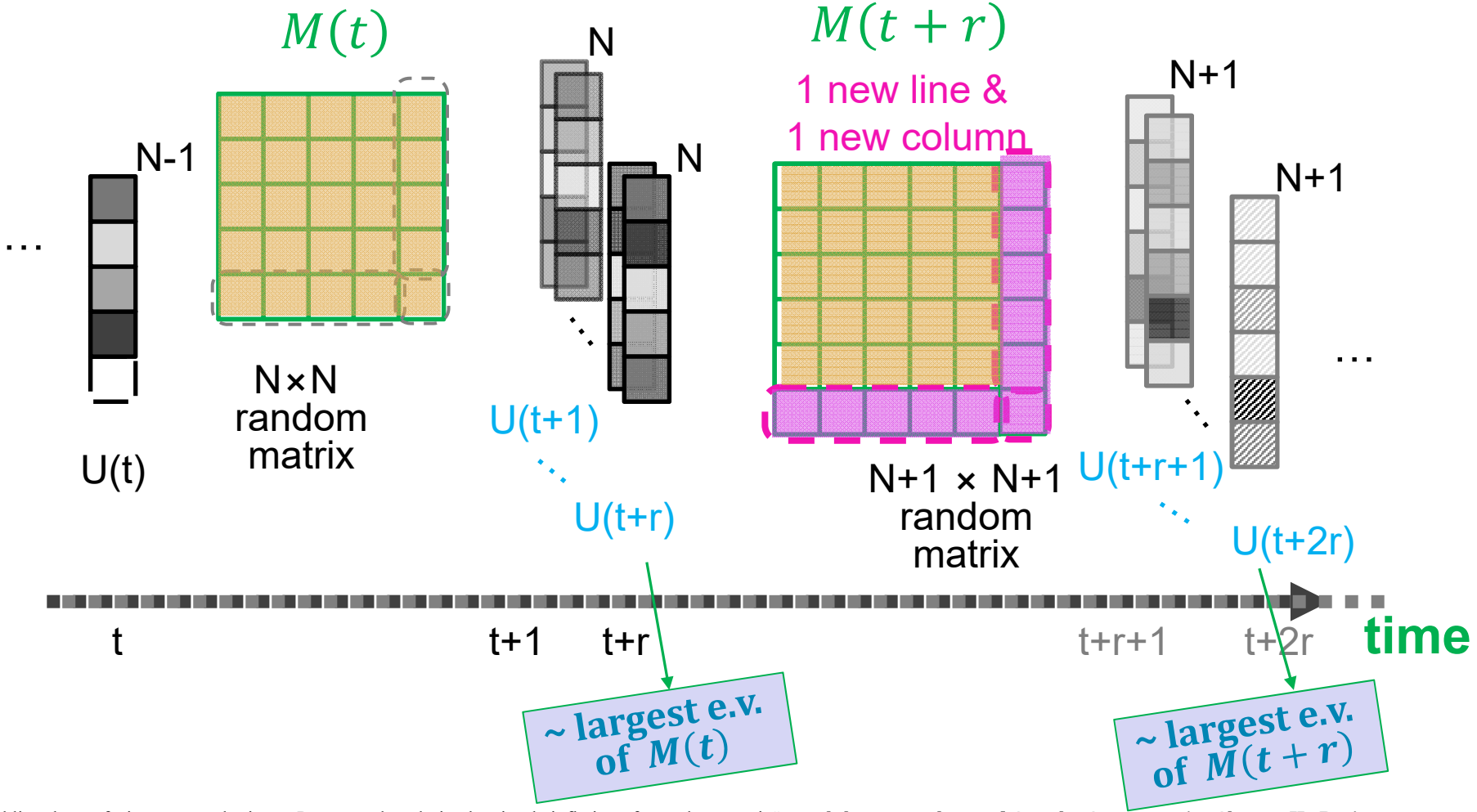
- **Novelty is discrete**
  - **→ ignoring the « noise »/brownian motion of Matrix(t)**
  - **→ Rather changing the size of space ($N \rightarrow N+1$) as the main trigger of changes.**
  - **→ So again, «** when does the tail (1 out of N) wags the dog ? **»**

Change by « inflating a matrix » (detailed)

(N+1)-th species/new product is related to all N prior ones.

$U(t+1) = M(t)U(t)$ with $M(t)$ an "inflating matrix"

U attracted [in r steps] to largest eigenvector… a "jumping" target !!



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Actual global growth ?

Unimportant (for the time being) :

any scaling of λ adds a factor $\exp(\lambda t)$, …

➔ Vector $U(t)$ can be normalized at each t.

t

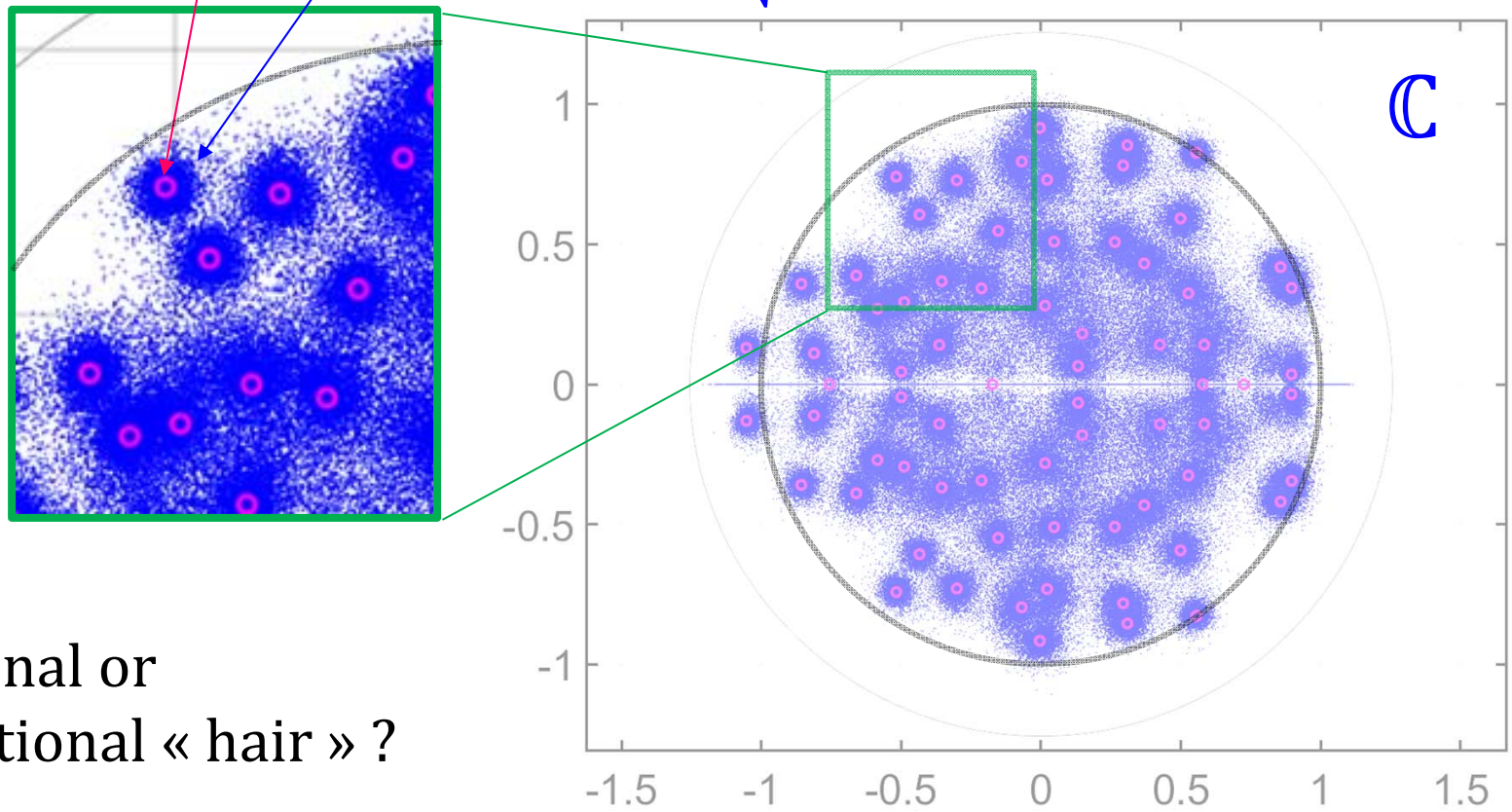
Eigenvalues of M & « random inflated M » (Ginibre N=70 example)

N=70

N+1=71

(full M, real,
spectral radius ρ(asymp)→1

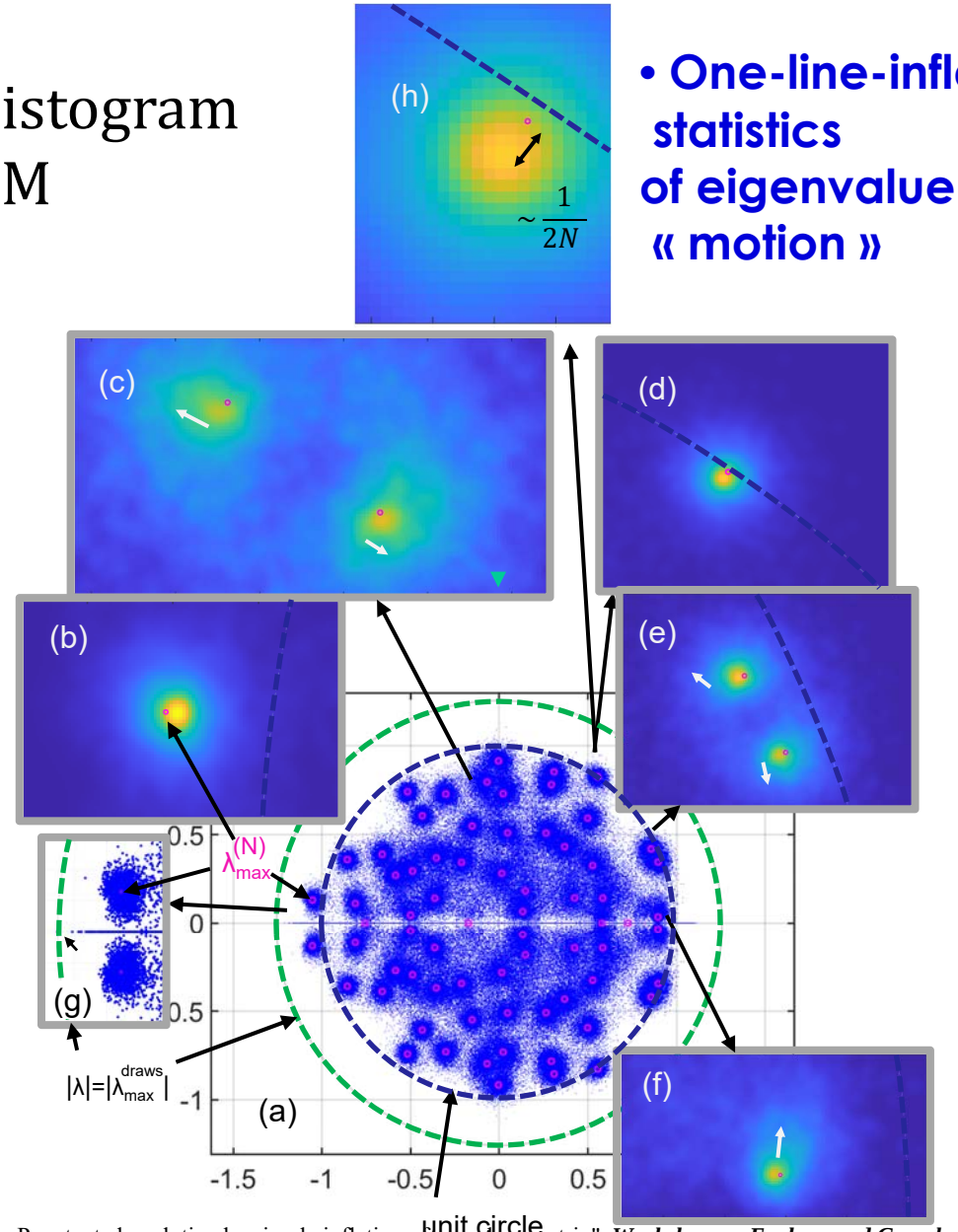
$\sqrt{\frac{N}{N+1}}$ normalized)



Directional or
undirectional « hair » ?

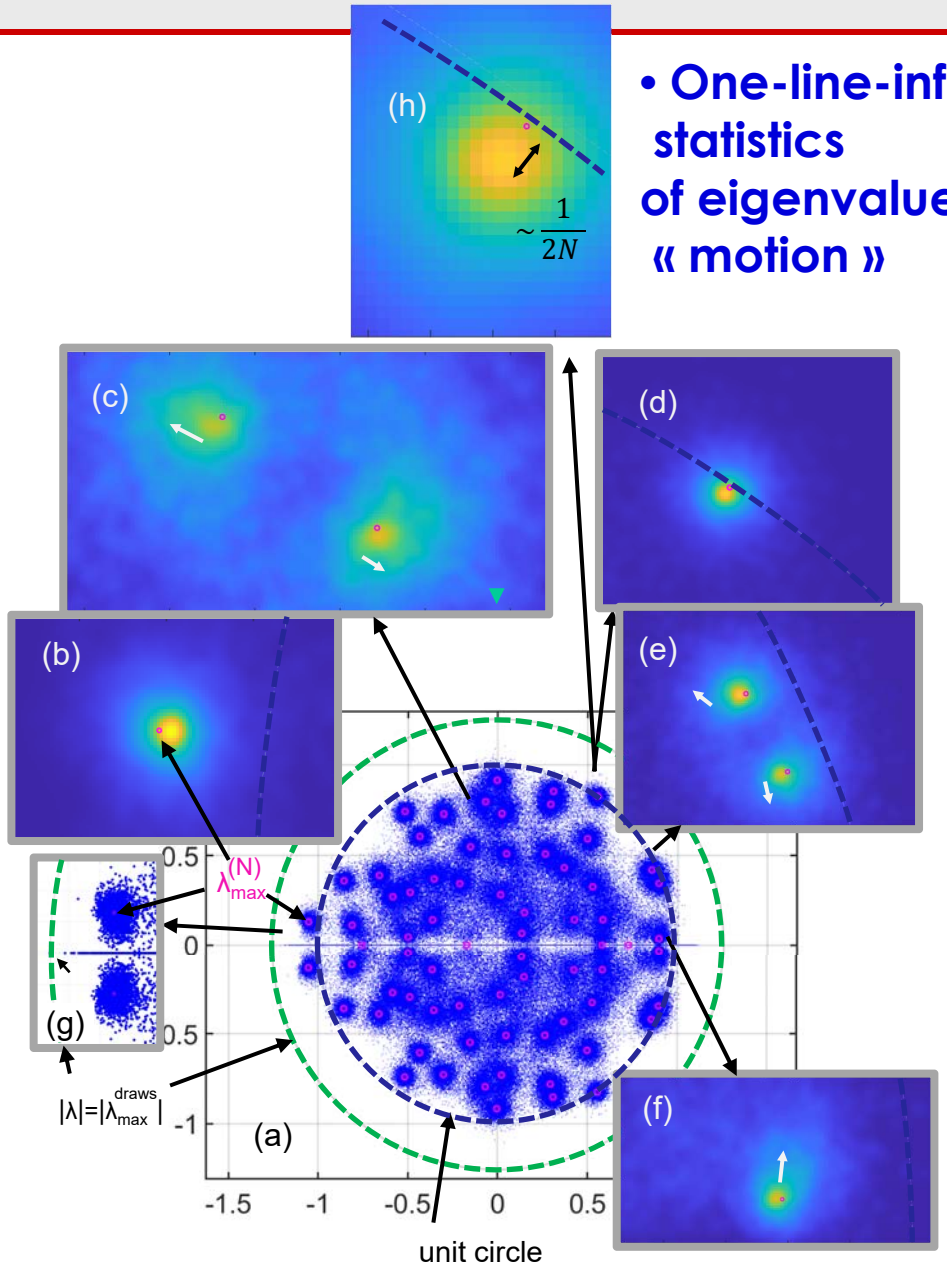
Moving target :Eigenvalue "motion" (Ginibre) upon inflation.

Coloured histogram of inflated-M ev' density



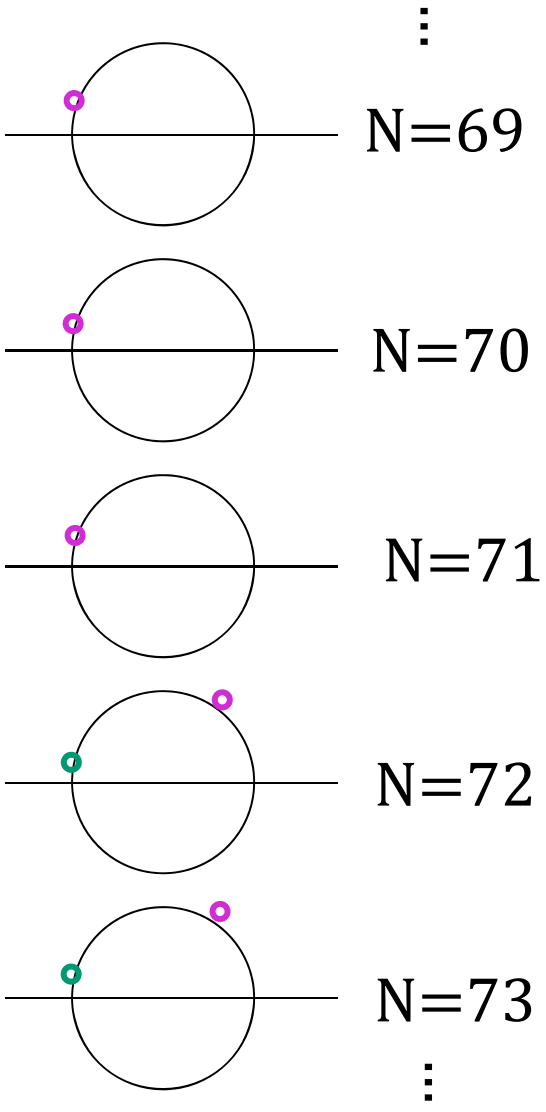
- **One-line-inflation statistics of eigenvalue « motion »**

Moving target :Eigenvalue "motion" (Ginibre) upon inflation.



- One-line-inflation statistics of eigenvalue « motion »

- Scenario for largest eigenvalue vs. N



"Iteration of vector" (à la Krylov) tracks largest eigenvalue

or eigenvalue.s

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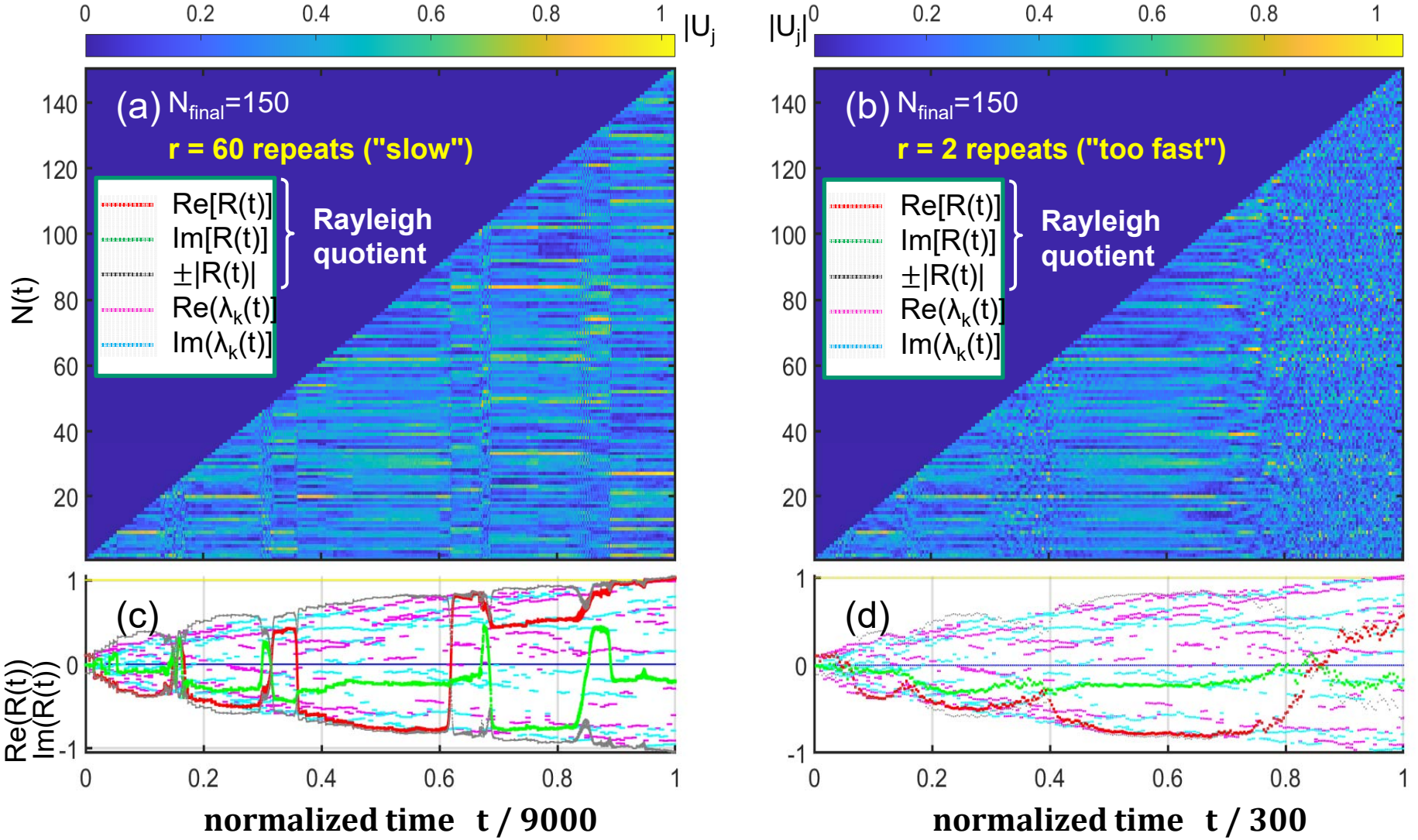
université PARIS-SACLAY

$$U(t+1) = M(t)U(t)$$

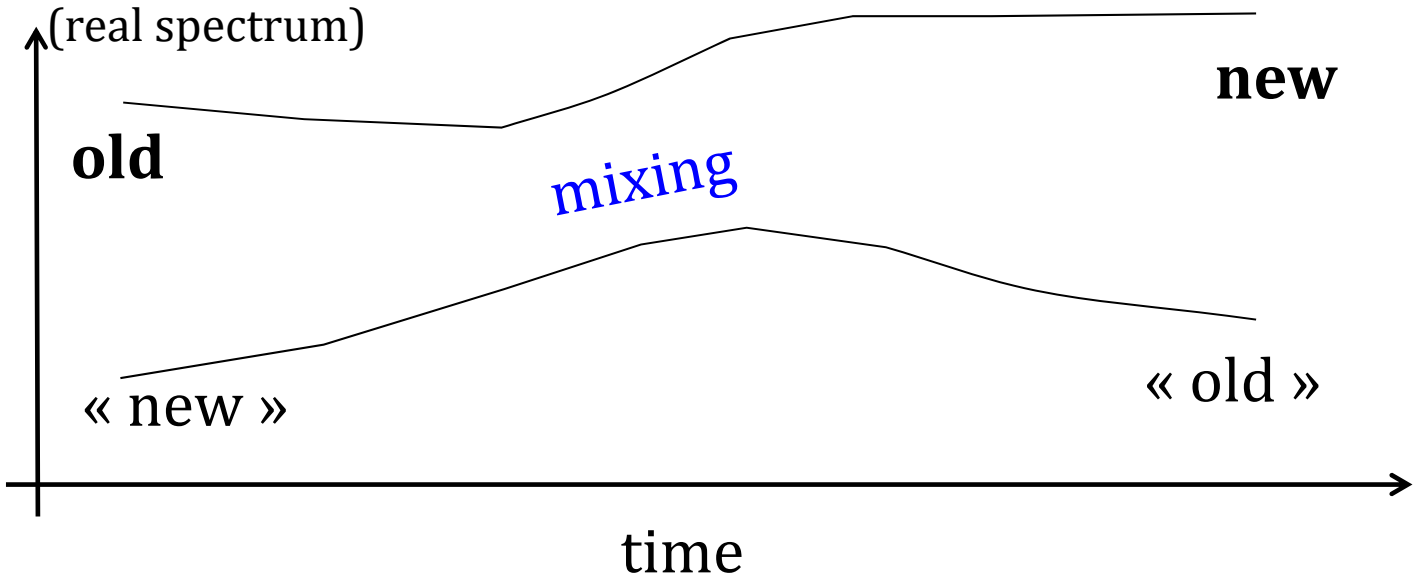
...if r ≳ 30 between two $N \to N+1$ inflations



Punctuation ⇔ Non hermitian growth

- It is possible for a Non-Hermitian vector to become « *new highest eigenvalue* » without « mixing » with old (« *prior highest* ») because complex eigenvalues can drift independently

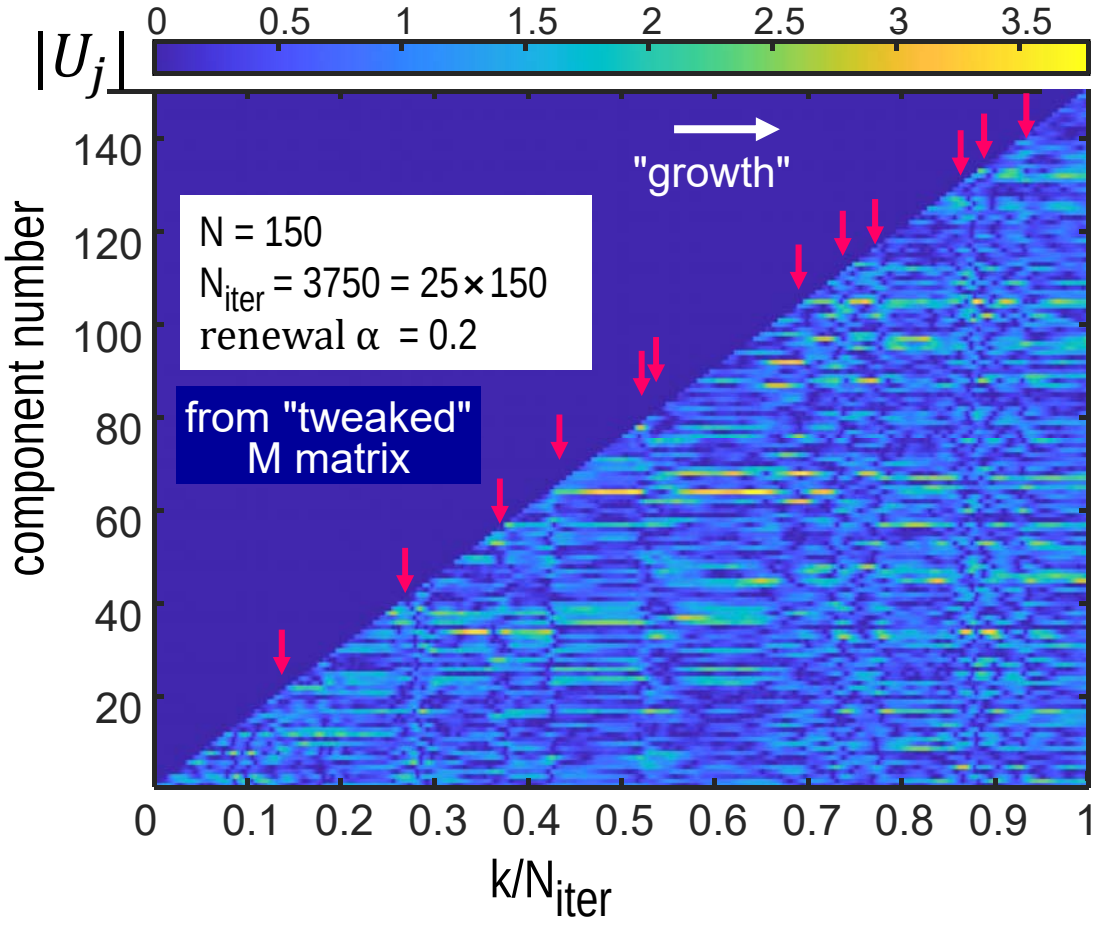
- For a Hermitian case (real spectrum), the « new vector » would *anticross* with the « old vector », → not a sudden competition, rather an incremental shift due to mixing.



Eigenvector size grows … with stasis and quakes !

(6/10)

(Real matrices : some fiddling needed to cope with paired eigenvalues)

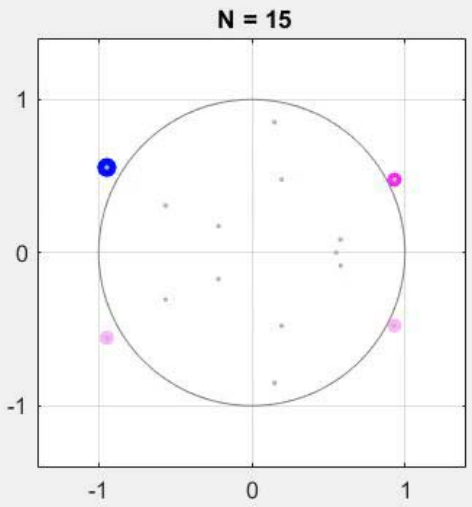


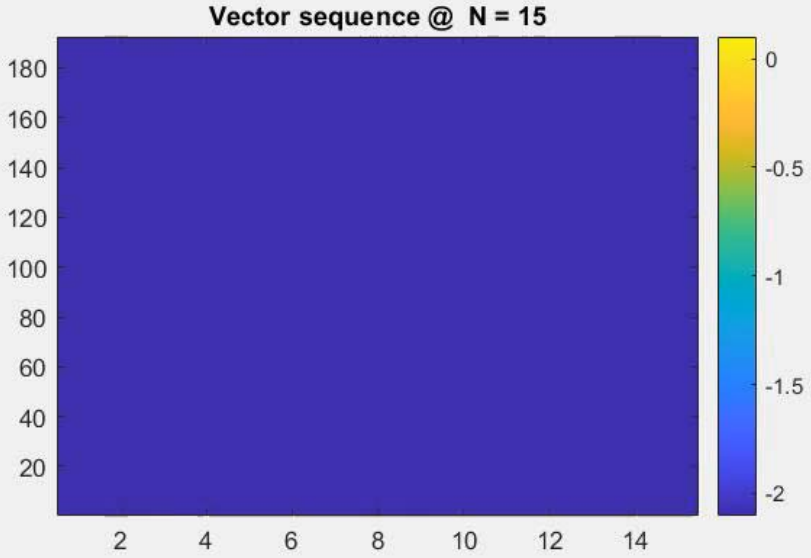
Color could represent abundance of species/goods…
(or log[Species])

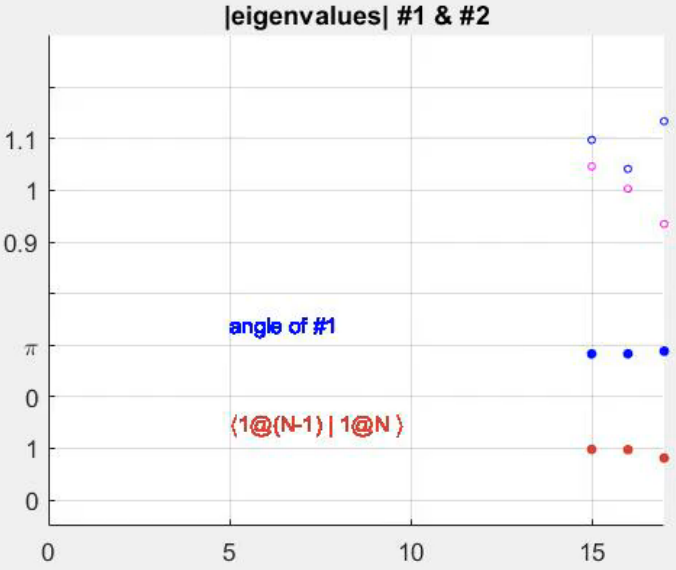
[2] H. Benisty, "Evolutionary behaviour …", J. Phys. Complex, vol.3, pp 025006, 2022.

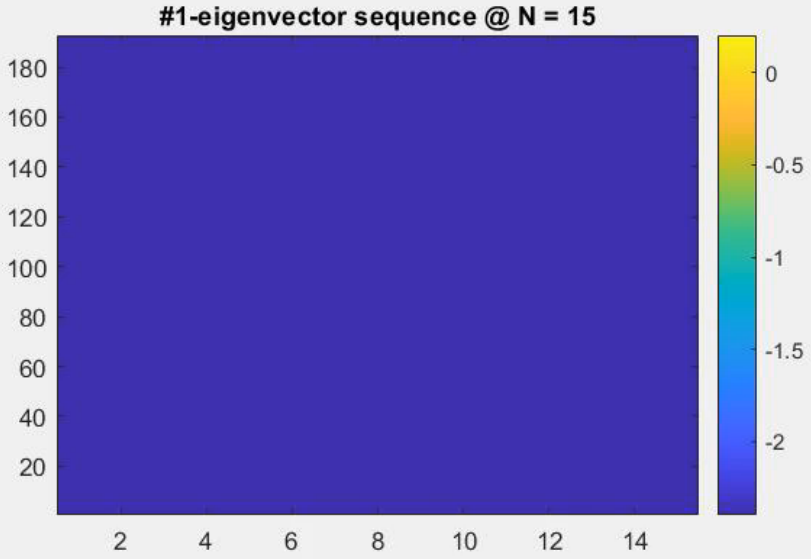
[3] H. Benisty, "Growth (…) Inflating Complex Random Matrices", Entropy vol. 25, pp.1507, 2023. https://doi.org/10.3390/e25111507

Movie 2021 version (avi)

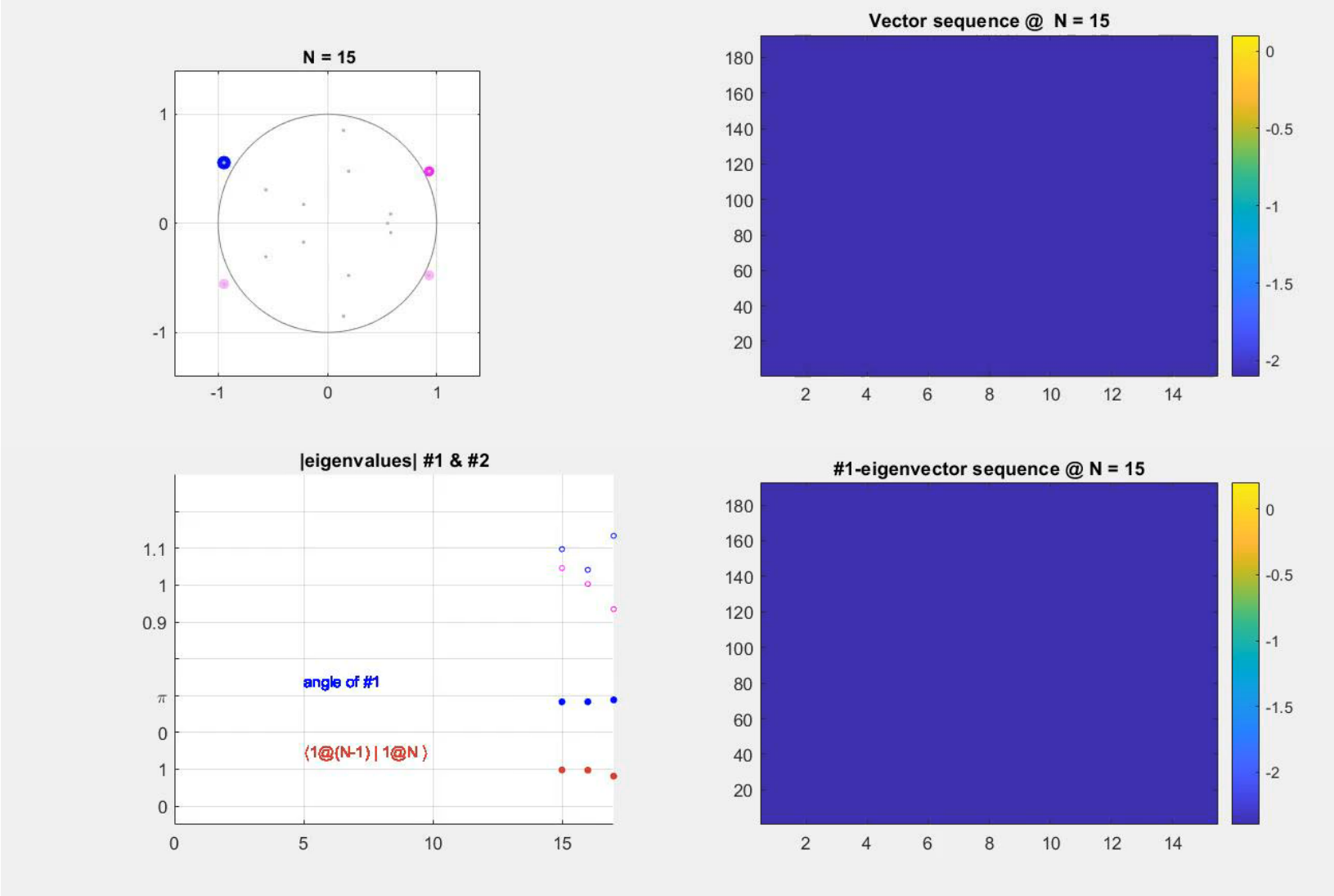




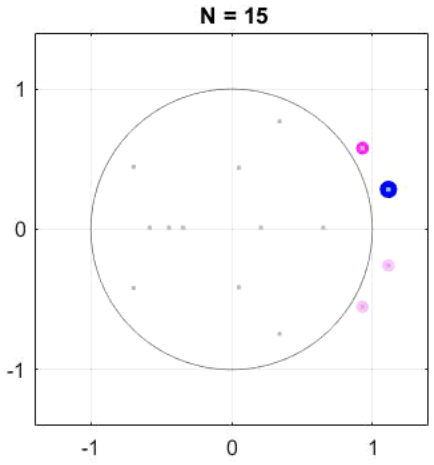


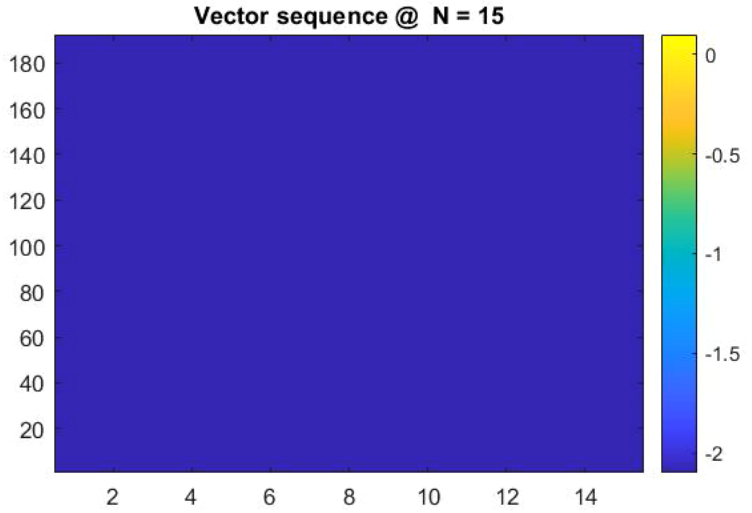


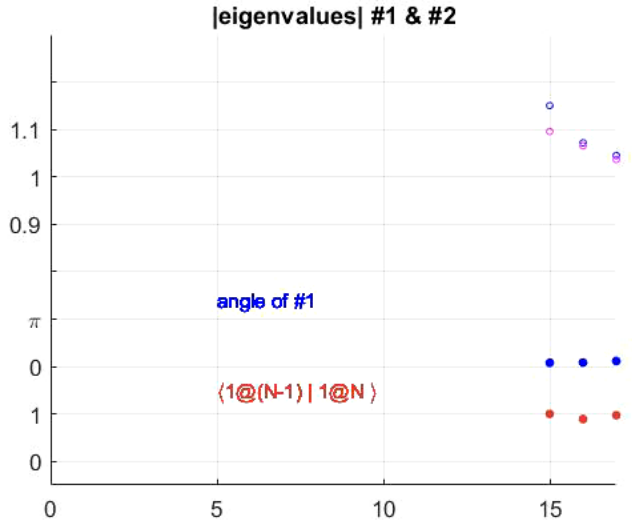
Movie 2021 version (mp4)

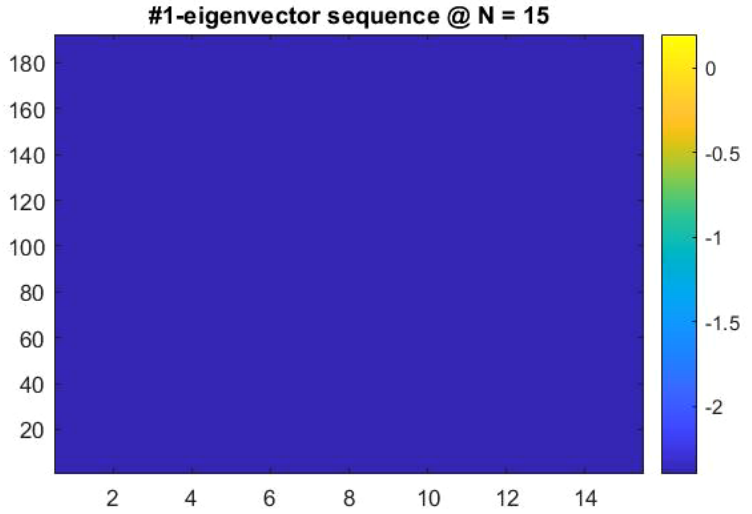


Movie 2024 version (avi)

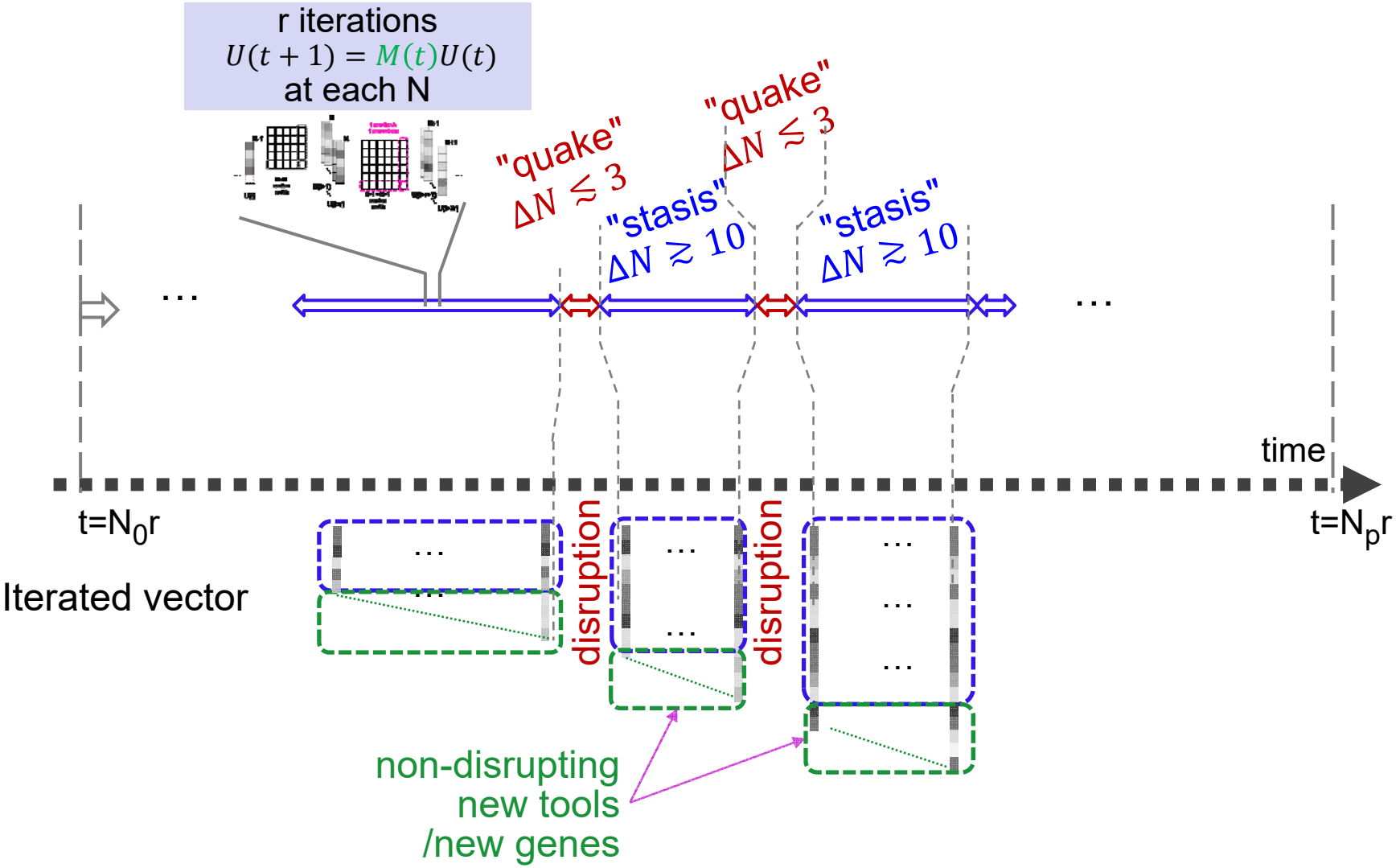








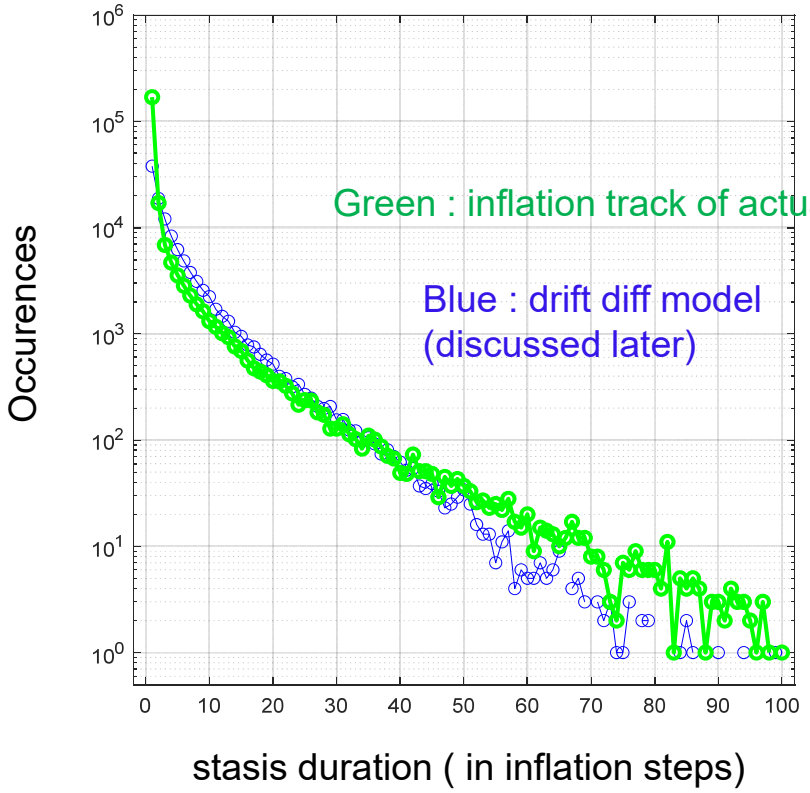
Wrap-up of timeline t and its scales



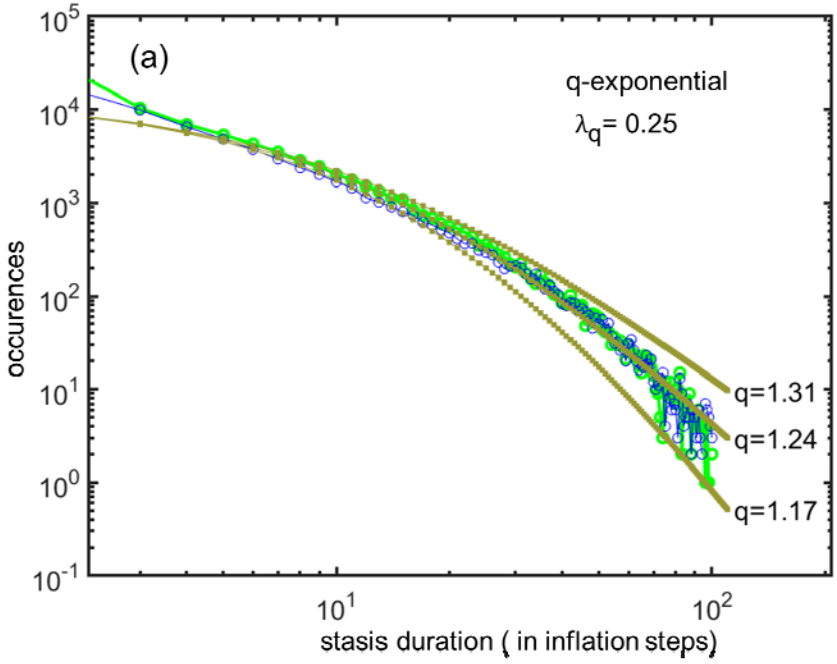
Stat' of stasis duration: q-exponential law ? (8/10)

(also found in paleontology !)

$$\mathrm{pdf}(x) = A_q\left[1 + (1-q)\left(-\lambda_q x\right)\right]^{\frac{1}{1-q}}$$

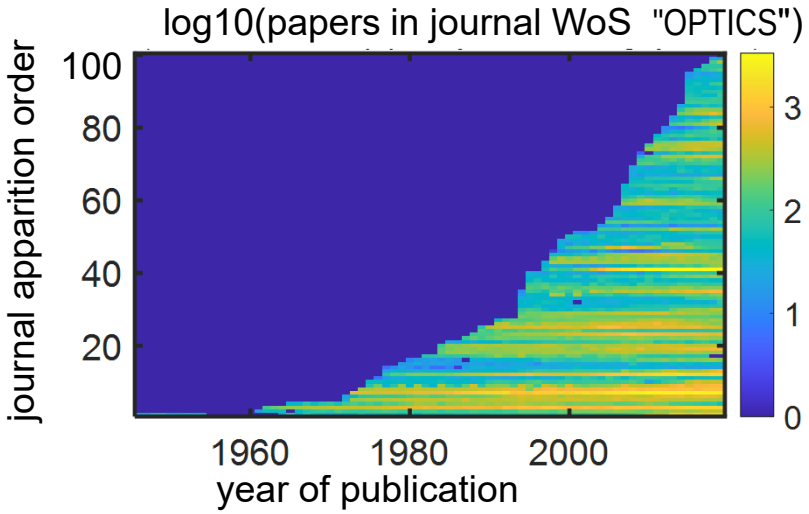


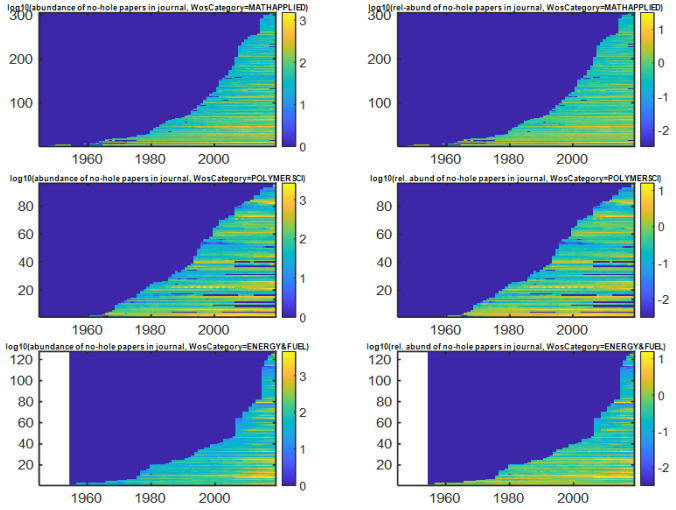
[30] T. Shimada, S, Yukawa, and N. Ito, "Life-span of families in fossil data forms q-exponential distribution," Int. J. Mod. Phys. C 14, 1267–1271 (2003)



Any actual « *growth-cum-profusion* » resembling our model ? (9/10)

Same as profusion of … scientific journals !





New vector with some « extinctions » after burst of new journals

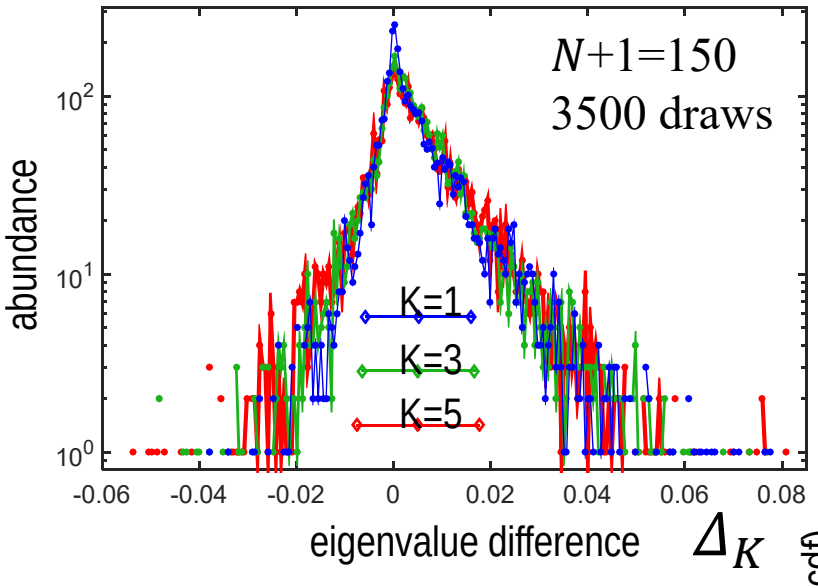
(looking carefully…)

OUTLINE

- **Growth, its extreme, its « noise »**
- My toy in photonics : « PT Symmetry » ⇔ Non-Hermitian H with real eigenvalues
- Geometric Brownian motion and non-ergodicity (2017)
- MacroEco models of PIB/Energy connexion, with « noise »
  (work with H. Bercegol, CEA/LIED)

- **Discrete perturbation and « punctuated growth »**
- Punctuated growth and instabilities in 2024
- Matrix inflation model : a « *disrupting or not disrupting* » eigenvalue race
- Drift-Diffusion « poor man's model » of eigenvalue race.

Exploration of eigenv's evolution @inflation



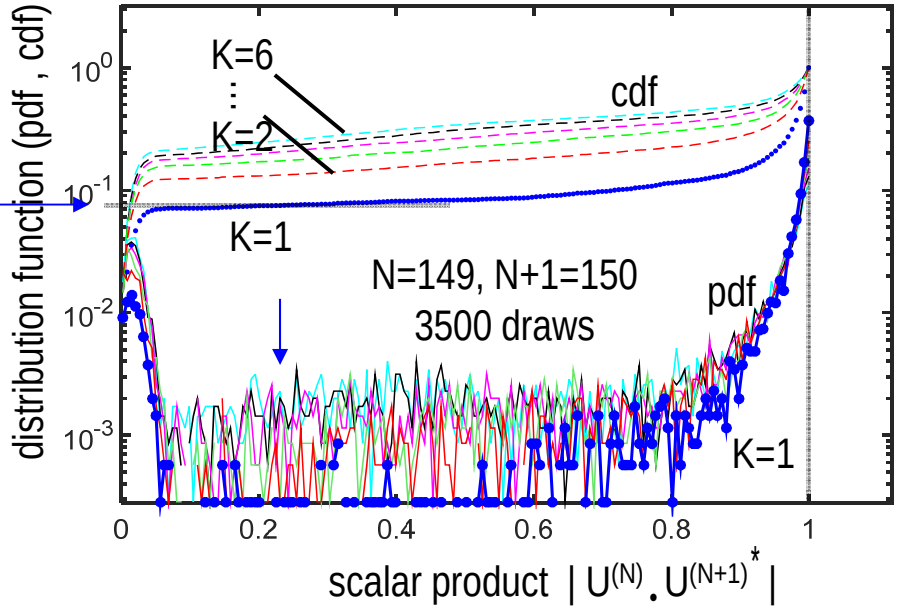
K = eigenvalue rank

K-th "eigenvalue shift » ?

$$\Delta_K = \sqrt{\frac{N}{N+1}}\lambda_K^{(N+1)} - \lambda_K^{(N)}$$

Two « half-Poisson » with slopes 2:1

pdf and cdf of hermitian scalar product
| <before | after> |
$|\langle U^{(N)}|U^{(N+1)}\rangle|$

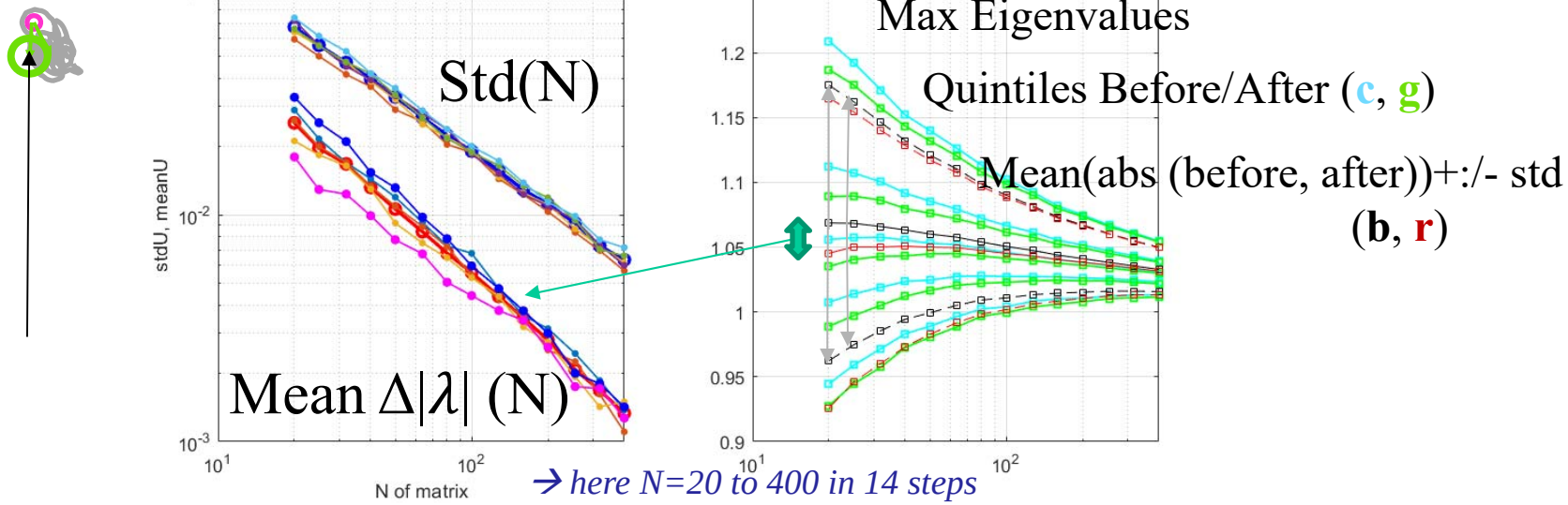


5%-7% « true change »
93-95% « moved »

Distribution of ev shift described by Mean(N,λ) & Std(N,λ)

Capturing two-particles $\lambda_{K=1,K=2}$ motion « agnostically »

Eigenv' ($\lambda_{K=1,K=2}$) evolution Before (size N) → After (size N+1)

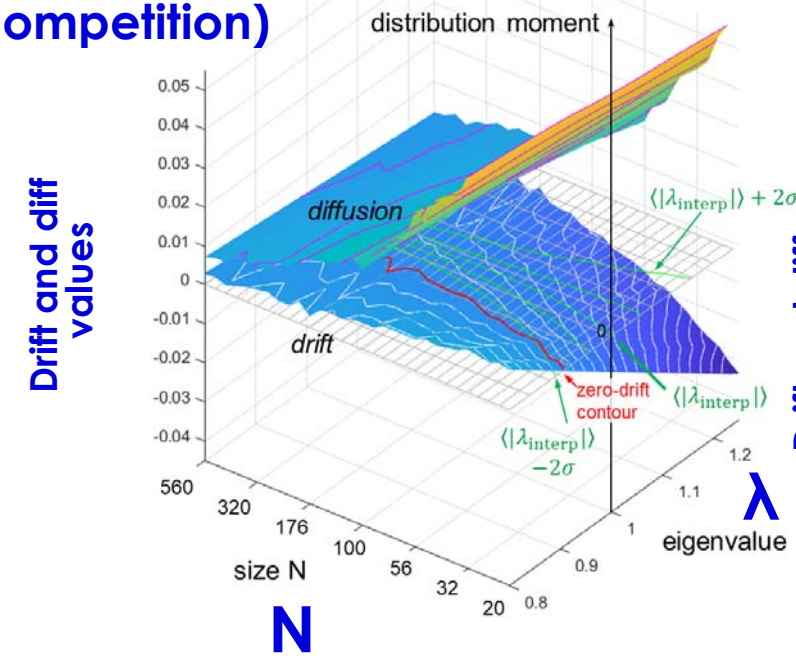


→ *here N=20 to 400 in 14 steps*

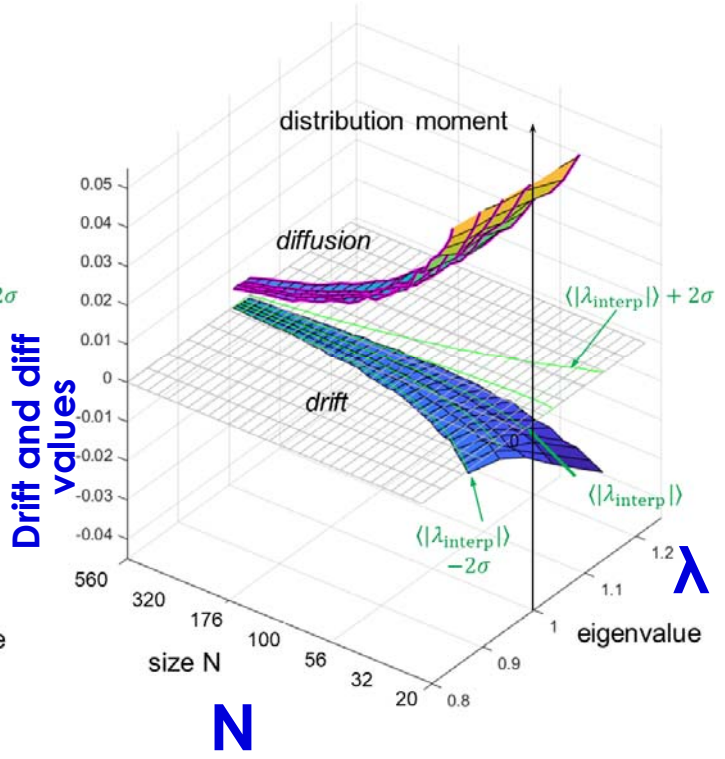
- Studied K=1 and K=2 together to capture « crossings » of two particles
- Studied variation for N=20 to 576 {20; 22; 25; 28; 32; 36}x{1 ; 2 ;4 ;8;16}
- Quadratic fit of mean $\lambda_{K=1,K=2}$ and std

Mapping of Mean(N,λ) & Std(N,λ)

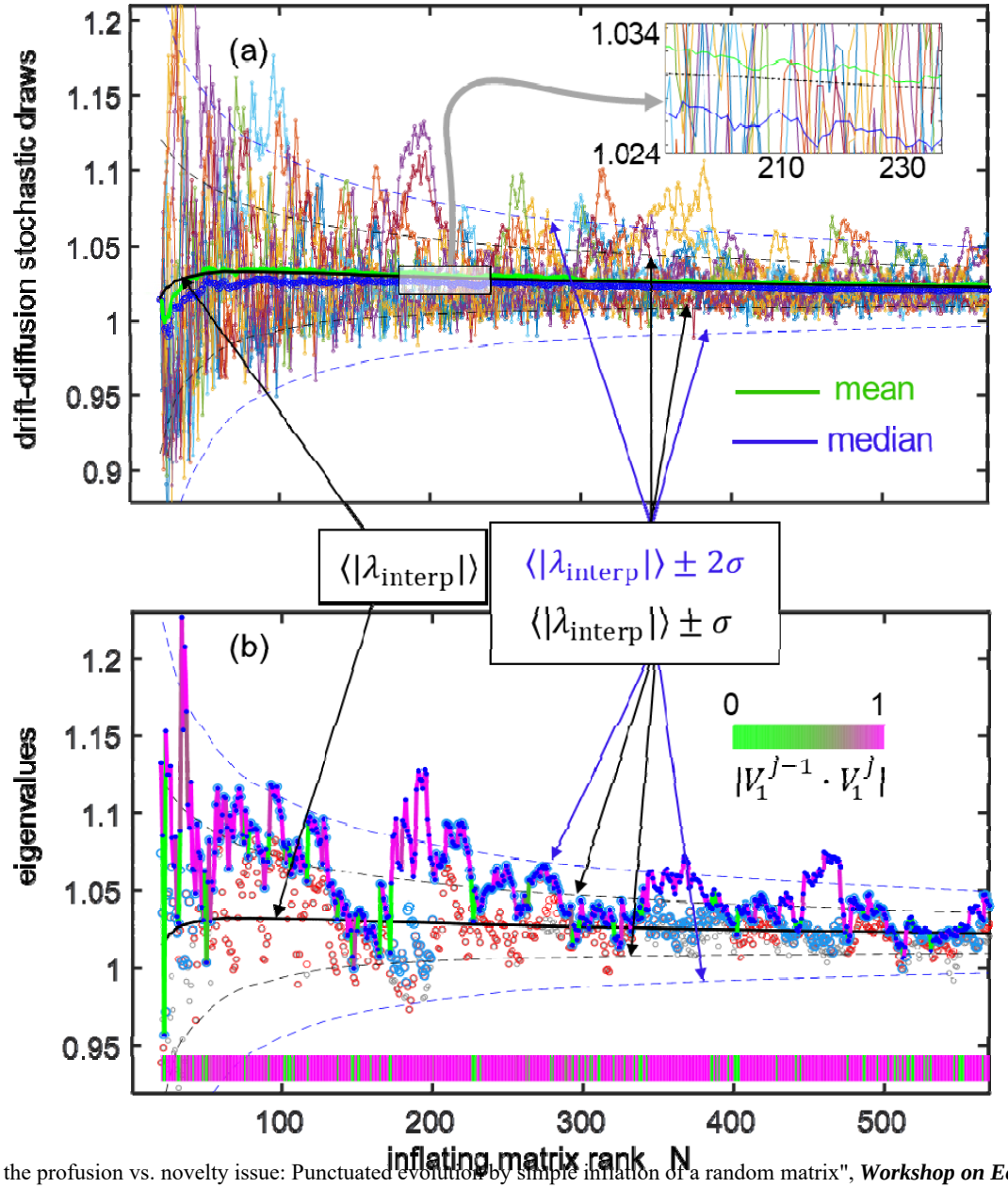
Interpolated map (to run a diffusion drift simulation and hope to test a two-ev's competition)



Data of interpolated map



« Ev's Racing » comparison



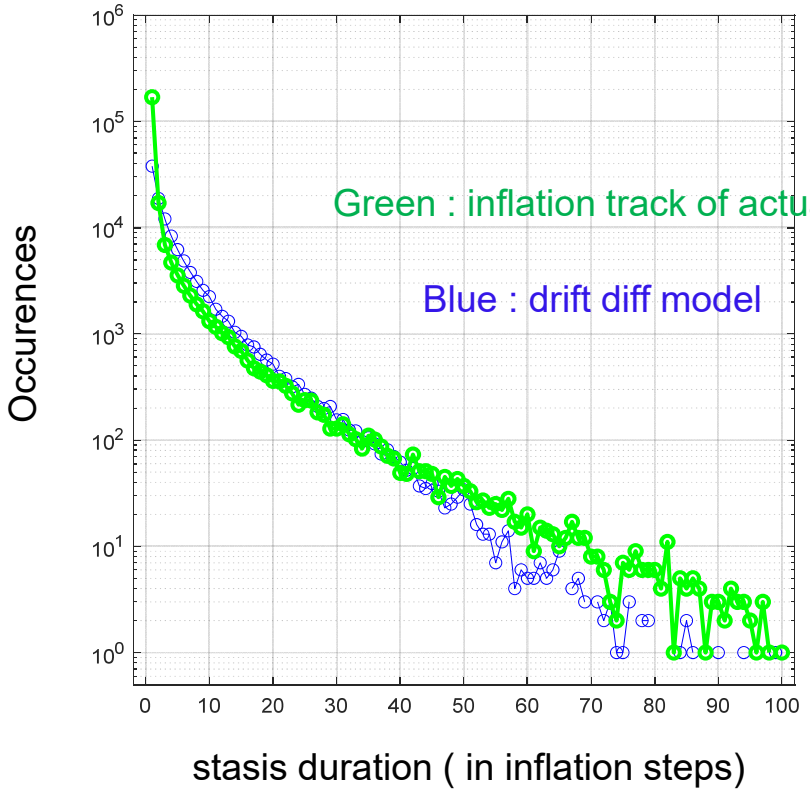
Ten examples of drift diffusion trajectories with $\langle\lambda\rangle$,$\langle\lambda\rangle \pm \sigma$, $\langle\lambda\rangle \pm 2\sigma$ of a large set

One example of « true matrix-inflation races »
+ same
+ signatures of vector change at « quakes »

Stat of stasis duration: q-exponential law ? (8/10)

(also found in paleontology !)

$$\mathrm{pdf}(x) = A_q\left[1 + (1-q)\left(-\lambda_q x\right)\right]^{\frac{1}{1-q}}$$



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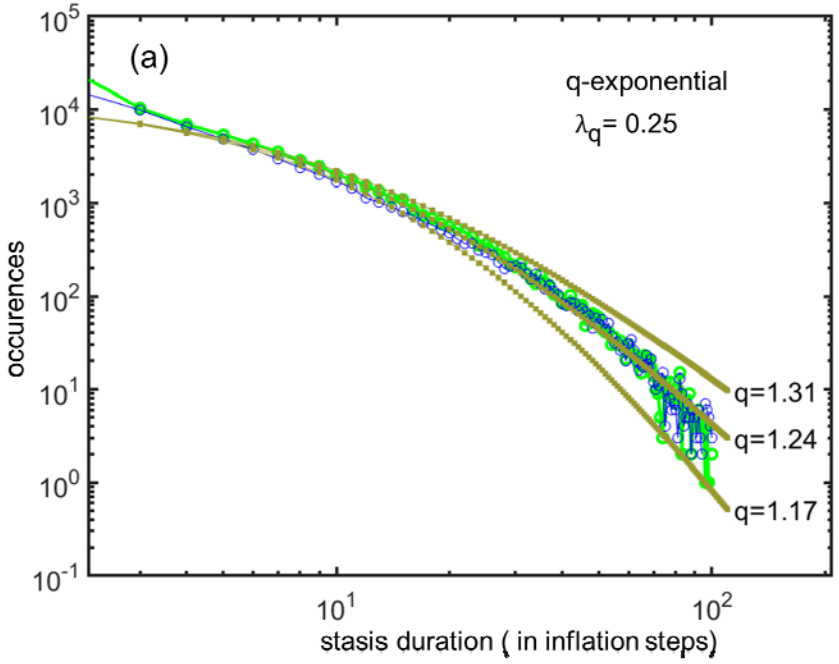
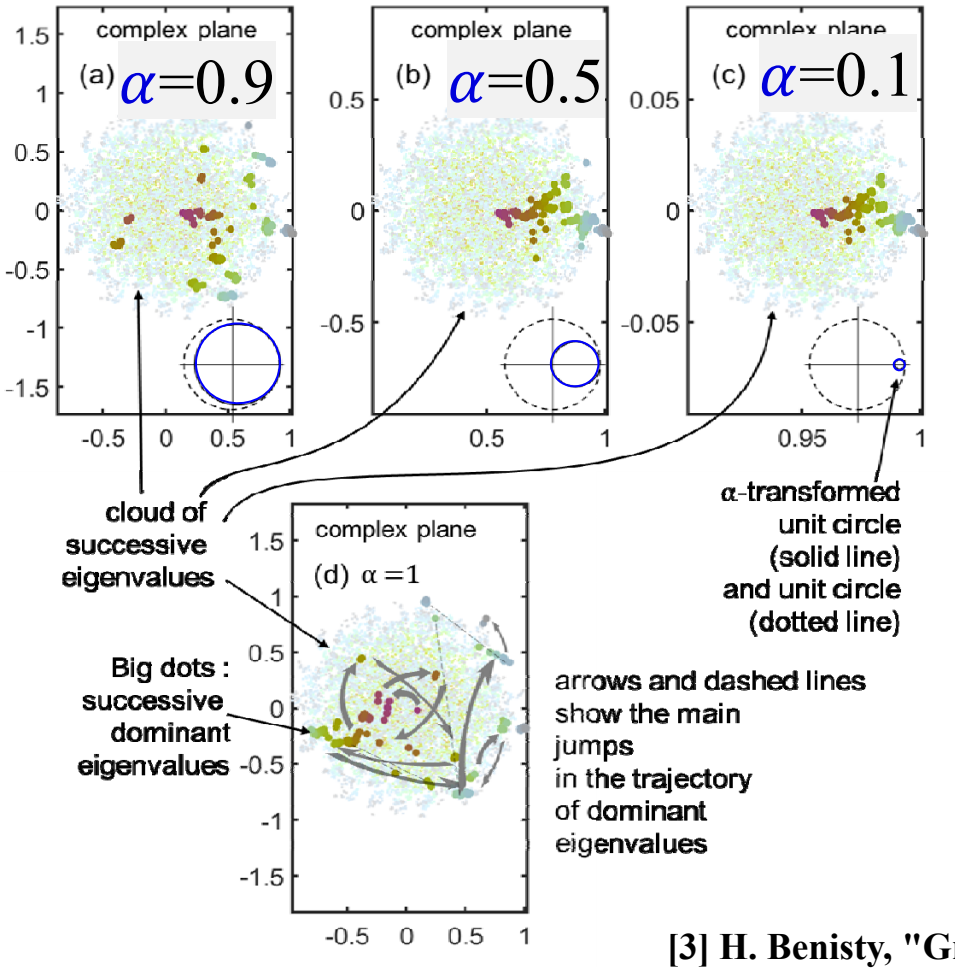
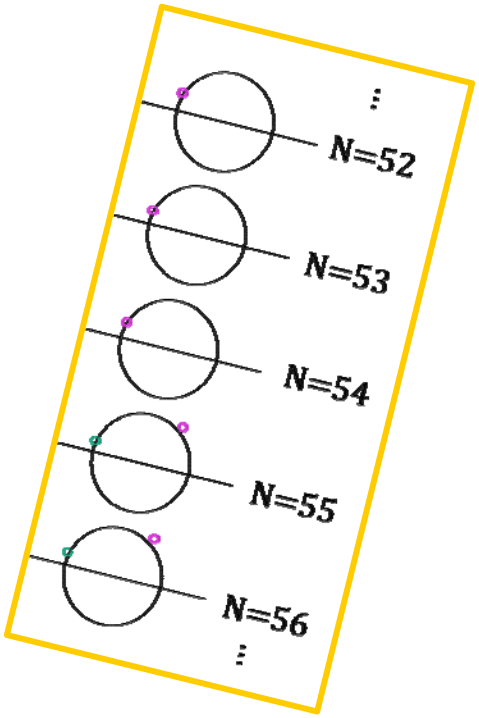


Subtleties when playing with the vector

For instance "partial renewal" of vector

$$U(t+1) = \alpha M(t) U(t) + (1-\alpha)\, U(t)$$



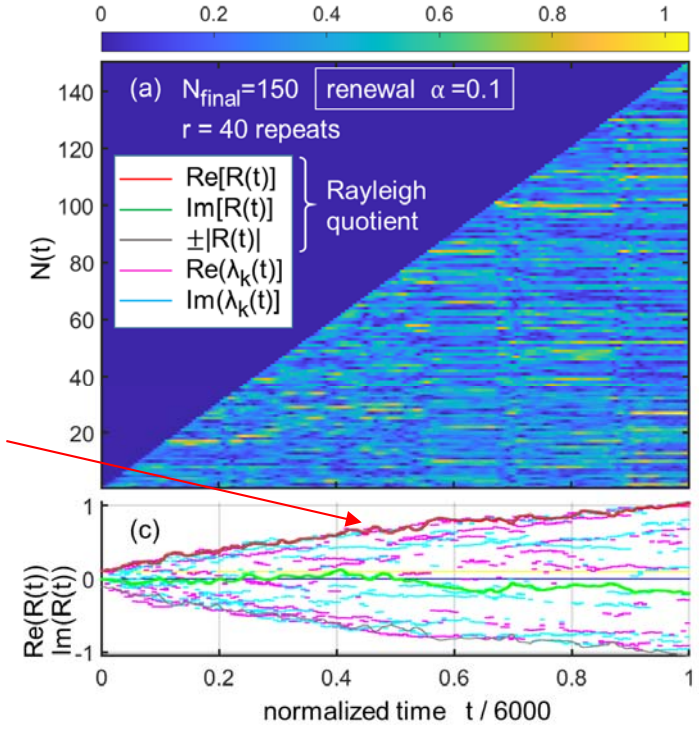


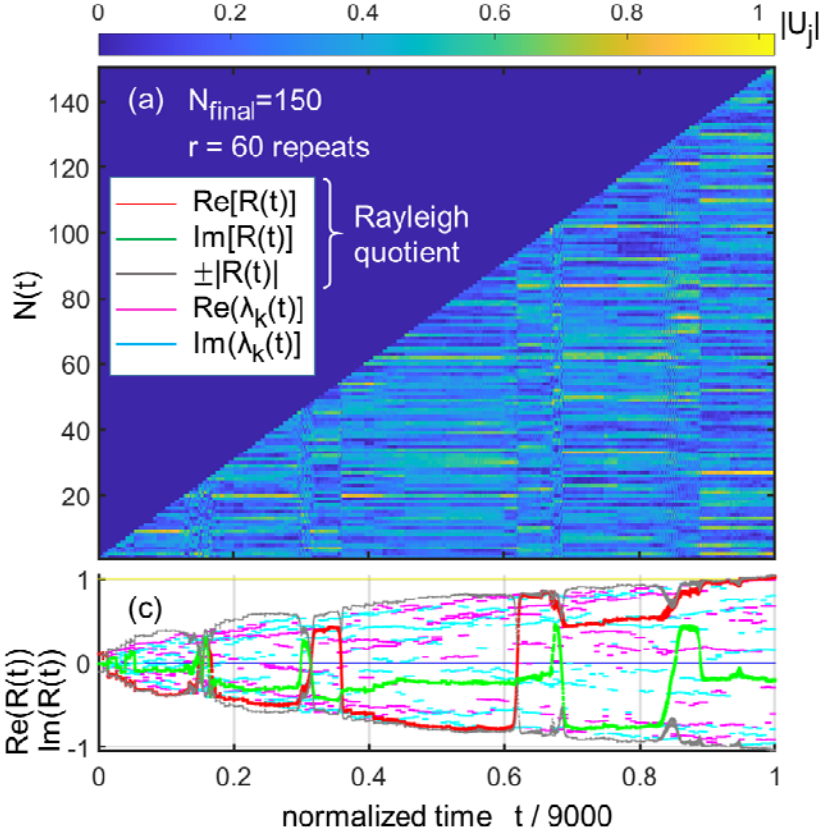
[3] H. Benisty, "Growth (…) Inflating Complex Random Matrices", Entropy vol. 25, pp.1507, 2023.

Still quaky but race is in reduced complex-plane circle

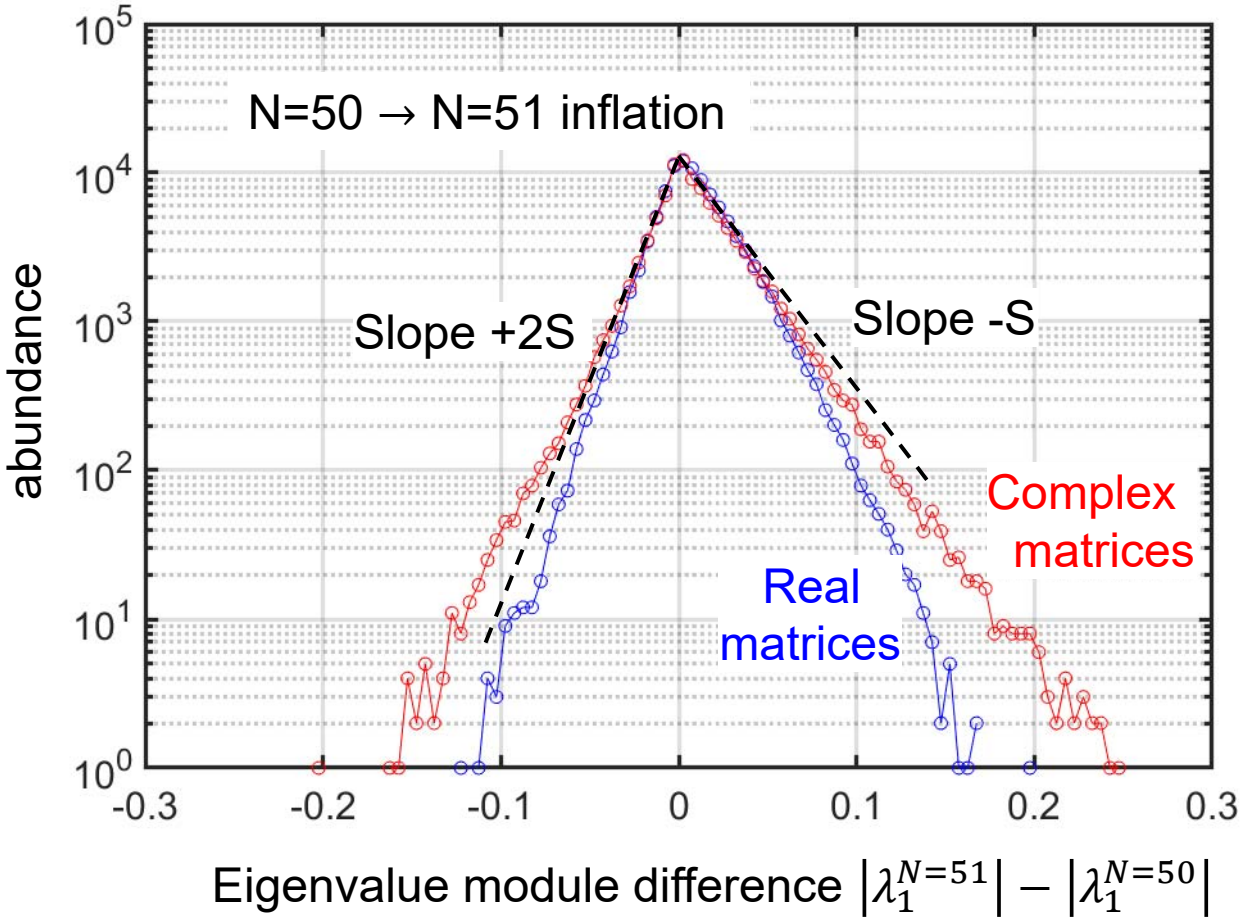
Case study of "partial renewal" of vector

$$U(t+1) = 0.1\, M(t)U(t) + 0.9U(t)$$

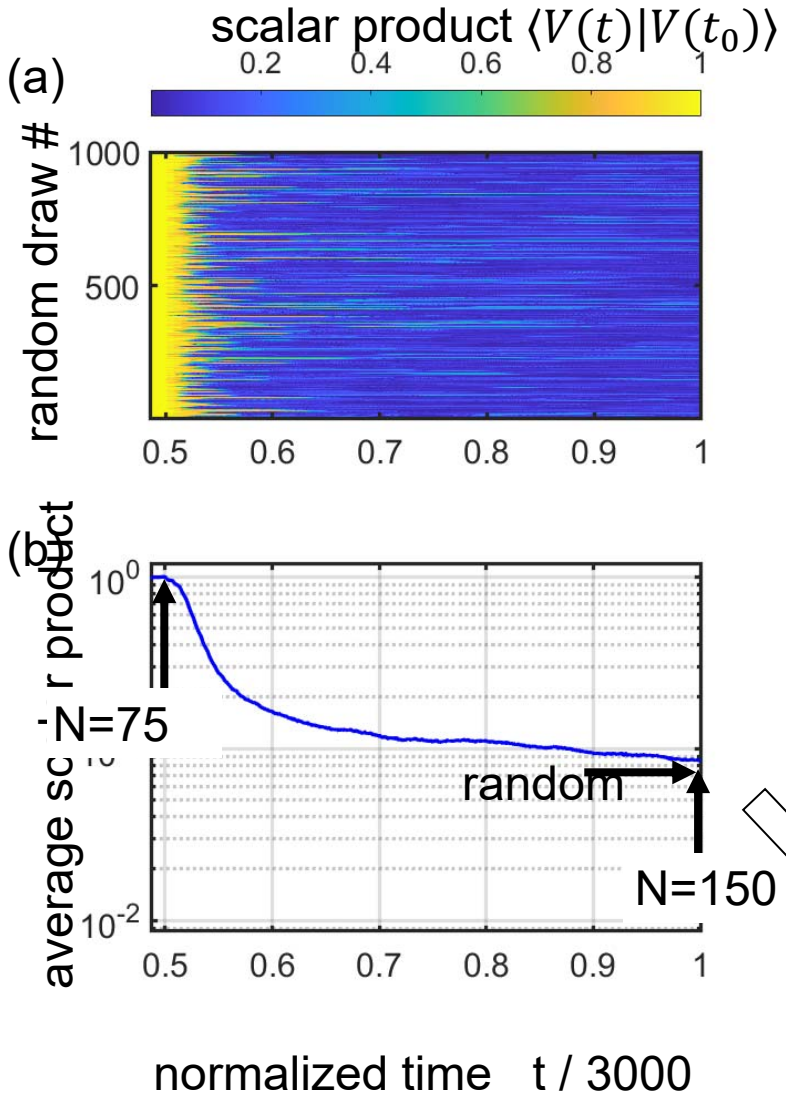




Ginibre M(t) vs . « full-complex » M(t) ?
Similar trend but more spread



The question of « HISTORY » , first glimpse



Take same « past » from N=15 to 75

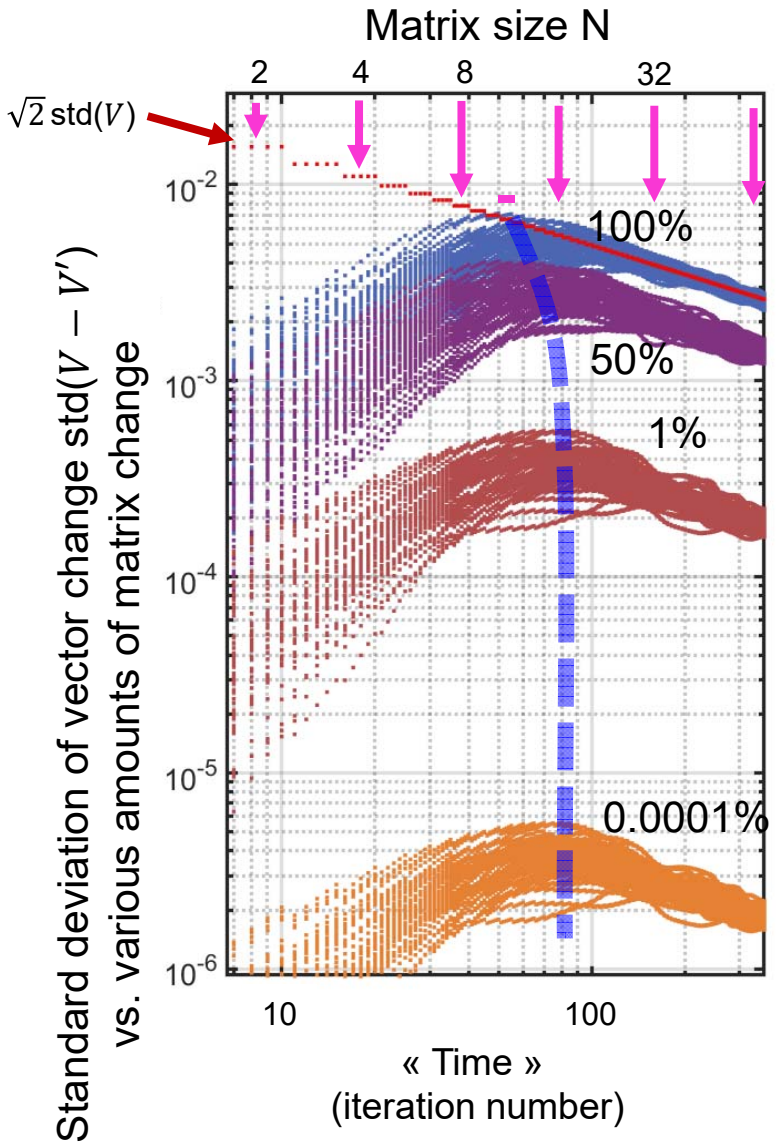
⇩

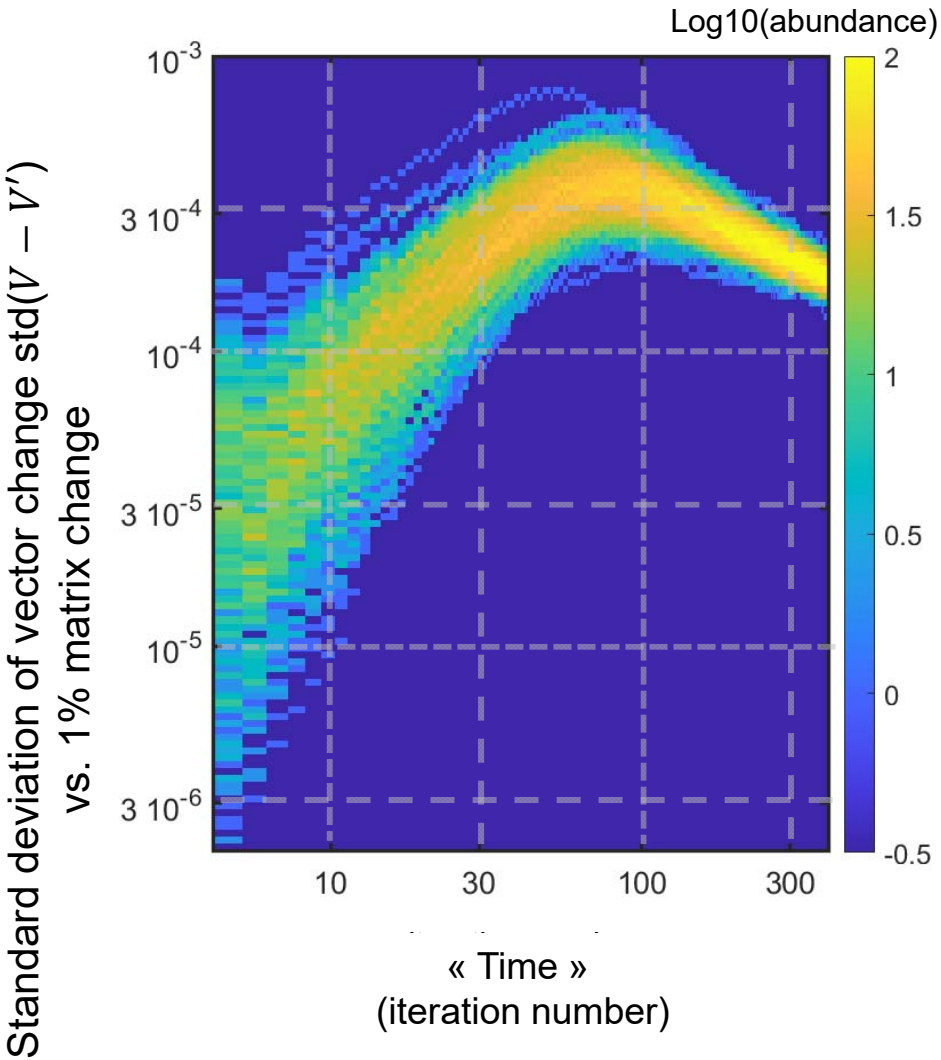
Test 1000 different futures
For N=75 to N=150 (random draws)

Check when « past » is forgotten on average

~ « after a few quakes »

A more « general » view to sensitivity to the RMT « M(t) »





Bolder « Evo-Rationale" for the interest of "RMT punctation"

- **Understanding where/when to hit within a complex process**

- **Difficulty to promote a smooth « gentle linear feedback » policy**

- **Rather (as companies do) be ready when « quakes triggered » (to push evolution in the intended direction)**

- **A model for growth/profusion could equally be turned into a model for flavours of « degrowth »**

Attrition of M(t) instead of growth/spawning : a « matrix deflation » process.

→ **RM models could help decide:**

What « cancellations" have the smallest impact?
« dynamic keystone species ? » (keystone goods ?)
… the jury is out !

Hervé Bercegol, Henri Benisty, An energy-based macroeconomic model …since 1820, Ecol. Econ., 192, 107253 (2022).

Henri Benisty, "… wealth distribution …inequality-induced crisis," PhysRev E.95.052307, (2017)

Acknowledgments

HB is grateful to Hervé Bercegol for support of an early version of these ideas. He also thanks Guillaume Martin, François Massol, Nicolas Pouyanne, Julien Randon-Furling and Thierry Huillet for several useful discussions in mathematics and evolutionary biology.

Conclusion/perspectives

- Punctuated growth as the result of the "not-so-random response" of eigenvectors/eigenvalues to the addition of one line and column to a Random Matrix

- Use a drift-diffusion model to reproduce the stasis-quakes dynamics. (drift and diffusion calibrated from agnostic RMT numerics)
- Glimpsed into non-Ginibre and « history » issues.

As an outsider :

- I would be glad to learn : other applications of RMT to punctuation
- Understanding the dynamics of LMEV (and eigenvector) with NH tools « (Condition number/Petermann factor / phase rigidity…) »
- Possibly understand the applicability of « cavity method » (Bethe's method) for such problems.
- Access large databases to track « profusion », eigenvectors and find some statistical properties (Rayleigh quotient, diffusion of vector)